International Mathematics TOURNAMENT OF THE TOWNS

Junior O-Level Paper

Spring 2014

- 1 [3] Each of given 100 numbers was increased by 1. Then each number was increased by 1 once more. Given that the first time the sum of the squares of the numbers was not changed find how this sum was changed the second time.
- 2 [4] Mother baked 15 pasties. She placed them on a round plate in a circular way: 7 with cabbage, 7 with meat and one with cherries in that exact order and put the plate into a microwave. All pasties look the same but Olga knows the order. However she doesn't know how the plate has been rotated in the microwave. She wants to eat a pasty with cherries. Can Olga eat her favourite pasty for sure if she is not allowed to try more than three other pasties?
- **3** [4] The entries of a 7×5 table are filled with numbers so that in each 2×3 rectangle (vertical or horizontal) the sum of numbers is 0. For 100 dollars Peter may choose any single entry and learn the number in it. What is the least amount of dollars he should spend in order to learn the total sum of numbers in the table for sure?
- **4** [5] Point *L* is marked on side *BC* of triangle *ABC* so that *AL* is twice as long as the median *CM*. Given that angle *ALC* is equal to 45° prove that *AL* is perpendicular to *CM*.
- **5** [6] Ali Baba and the 40 thieves want to cross Bosporus strait. They made a line so that any two people standing next to each other are friends. Ali Baba is the first; he is also a friend with the thief next to his neighbour. There is a single boat that can carry 2 or 3 people and these people must be friends. Can Ali Baba and the 40 thieves always cross the strait if a single person cannot sail?