International Mathematics

35th TOURNAMENT OF THE TOWNS

Junior A-Level Paper **Spring 2014**

1 [3] During Christmas party Santa handed out to the children 47 chocolates and 74 marmalades. Each girl got 1 more chocolate than each boy but each boy got 1 more marmalade than each girl. What was the number of the children?

2 [5] Peter marks several cells on a 5×5 board. Basil wins if he can cover all marked cells with three-cell corners. The corners must be inside the board and not overlap. What is the least number of cells Peter should mark to prevent Basil from winning? (Cells of the corners must coincide with the cells of the board).

3 [6] A square table is covered with a square cloth (may be of a different size) without folds and wrinkles. All corners of the table are left uncovered and all four hanging parts are triangular. Given that two adjacent hanging parts are equal prove that two other parts are also equal.

4 [7] The King called two wizards. He ordered First Wizard to write down 100 positive integers (not necessarily distinct) on cards without revealing them to Second Wizard. Second Wizard must correctly determine all these integers, otherwise both wizards will lose their heads. First Wizard is allowed to provide Second Wizard with a list of distinct integers, each of which is either one of the integers on the cards or a sum of some of these integers. He is not allowed to tell which integers are on the cards and which integers are their sums. If Second Wizard correctly determines all 100 integers the King tears as many hairs from each wizard's beard as the number of integers in the list given to Second Wizard. What is the minimal number of hairs each wizard should sacrifice to stay alive?

5 [7] There are several white and black points. Every white point is connected with every black point by a segment. Each segment is equipped with a positive integer. For any closed circuit the product of the integers on the segments passed in the direction from white to black point is equal to the product of the integers on the segments passed in the opposite direction. Can one always place the integer at each point so that the integer on each segment is the product of the integers at its ends?

6 [9] A $3\times3\times3$ cube is made of $1\times1\times1$ cubes glued together. What is the maximal number of small cubes one can remove so the remaining solid has the following features:

1) Projection of this solid on each face of the original cube is a 3×3 square;

2) The resulting solid remains face-connected (from each small cube one can reach any other small cube along a chain of consecutive cubes with common faces).

7 [9] Points A_1, A_2, …, A_{10} are marked on a circle clockwise. It is known that these points can be divided into pairs of points symmetric with respect to the centre of the circle. Initially at each marked point there was a grasshopper. Every minute one of the grasshoppers jumps over its neighbour along the circle so that the resulting distance between them doesn't change. It is not allowed to jump over any other grasshopper and to land at a point already occupied. It occurred that at some moment nine grasshoppers were found at points A_1, A_2, …, A_9 and the tenth grasshopper was on arc $A_9A_{10}A_1$. Is it necessarily true that this grasshopper was exactly at point A_{10}?