Senior O-Level Paper   **Spring 2011.**

1. The faces of a convex polyhedron are similar triangles. Prove that this polyhedron has two pairs of congruent faces.

2. Worms grow at the rate of 1 metre per hour. When they reach their maximum length of 1 metre, they stop growing. A full-grown worm may be dissected into two new worms of arbitrary lengths totalling 1 metre. Starting with 1 full-grown worm, can one obtain 10 full-grown worms in less than 1 hour?

3. Along a circle are 100 white points. An integer k is given, where $2 \leq k \leq 50$. In each move, we choose a block of k adjacent points such that the first and the last are white, and we paint both of them black. For which values of k is it possible for us to paint all 100 points black after 50 moves?

4. Four perpendiculars are drawn from four vertices of a convex pentagon to the opposite sides. If these four lines pass through the same point, prove that the perpendicular from the fifth vertex to the opposite side also passes through this point.

5. In a country, there are 100 towns. Some pairs of towns are joined by roads. The roads do not intersect one another except meeting at towns. It is possible to go from any town to any other town by road. Prove that it is possible to pave some of the roads so that the number of paved roads at each town is odd.

Note: The problems are worth 3, 4, 4, 5 and 5 points respectively.