

**International Mathematics
TOURNAMENT OF THE TOWNS**

Senior O-Level Paper

Fall 2011.

1. Several guests at a round table are eating from a basket containing 2011 berries. Going in clockwise direction, each guest has eaten either twice as many berries as or six fewer berries than the next guest. Prove that not all the berries have been eaten.
2. Peter buys a lottery ticket on which he enters an n -digit number, none of the digits being 0. On the draw date, the lottery administrators will reveal an $n \times n$ table, each cell containing one of the digits from 1 to 9. A ticket wins a prize if it does *not* match any row or column of this table, read in either direction. Peter wants to bribe the administrators to reveal the digits on some cells chosen by Peter, so that Peter can guarantee to have a winning ticket. What is the minimum number of digits Peter has to know?
3. In a convex quadrilateral $ABCD$, $AB = 10$, $BC = 14$, $CD = 11$ and $DA = 5$. Determine the angle between its diagonals.
4. Positive integers $a < b < c$ are such that $b + a$ is a multiple of $b - a$ and $c + b$ is a multiple of $c - b$. If a is a 2011-digit number and b is a 2012-digit number, exactly how many digits does c have?
5. In the plane are 10 lines in general position, which means that no 2 are parallel and no 3 are concurrent. Where 2 lines intersect, we measure the smaller of the two angles formed between them. What is the maximum value of the sum of the measures of these 45 angles?

Note: The problems are worth 3, 4, 4, 4 and 5 points respectively.