INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS

Senior A-Level Paper, Fall 2011.

Grades 11 – 12

(The result is computed from the three problems with the highest scores, the scores for the individual parts of a single problem are summed.)

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points	problems
4	1. Pete has marked several (three or more) points in the plane such that all distances between them are different. A pair of marked points $A; B$ will be called unusual if A is the furthest marked point from B, and B is the nearest marked point to A (apart from A itself). What is the largest possible number of unusual pairs that Pete can obtain?
4	2. Given that $0 < a, b, c, d < 1$ and $abcd = (1 - a)(1 - b)(1 - c)(1 - d)$, prove that $$(a + b + c + d) - (a + c)(b + d) \geq 1.$$
5	3. In triangle ABC, points A_1, B_1, C_1 are bases of altitudes from vertices A, B, C, and points C_A, C_B are the projections of C_1 to AC and BC respectively. Prove that line C_AC_B bisects the segments C_1A_1 and C_1B_1.
	4. Does there exist a convex N-gon such that all its sides are equal and all vertices belong to the parabola $y = x^2$ for
3	a) $N = 2011$;
4	b) $N = 2012$?
7	5. We will call a positive integer *good* if all its digits are nonzero. A good integer will be called *special* if it has at least k digits and their values strictly increase from left to right. Let a good integer be given. At each move, one may either add some special integer to its digital expression from the left or from the right, or insert a special integer between any two its digits, or remove a special number from its digital expression.What is the largest k such that any good integer can be turned into any other good integer by such moves?
7	6. Prove that the integer $1^1 + 3^3 + 5^5 + \ldots + (2^n - 1)^{2^n - 1}$ is a multiple of 2^n but not a multiple of 2^{n+1}.
9	7. 100 red points divide a blue circle into 100 arcs such that their lengths are all positive integers from 1 to 100 in an arbitrary order. Prove that there exist two perpendicular chords with red endpoints.