International Mathematics
TOURNAMENT OF THE TOWNS

Senior O-Level Paper   **Spring 2010**

1 [3] 2010 ships deliver bananas, lemons and pineapples from South America to Russia. The total number of bananas on each ship equals the number of lemons on all other ships combined, while the total number of lemons on each ship equals the total number of pineapples on all other ships combined. Prove that the total number of fruits is a multiple of 31.

2 [4] Let $f(x)$ be a function such that every straight line has the same number of intersection points with the graph $y = f(x)$ and with the graph $y = x^2$. Prove that $f(x) = x^2$.

3 [5] Is it possible to cover the surface of a regular octahedron by several regular hexagons without gaps and overlaps? (A regular octahedron has 6 vertices, each face is an equilateral triangle, each vertex belongs to 4 faces.)

4 [5] Assume that $P(x)$ is a polynomial with integer nonnegative coefficients, different from constant. Baron Münchausen claims that he can restore $P(x)$ provided he knows the values of $P(2)$ and $P(P(2))$ only. Is the baron's claim valid?

5 [6] A needle (a segment) lies on a plane. One can rotate it $45°$ around any of its endpoints. Is it possible that after several rotations the needle returns to initial position with the endpoints interchanged?