International Mathematics
TOURNAMENT OF THE TOWNS

Senior O-Level Paper **Fall 2009.**[1]

1. A 7-digit passcode is called good if all digits are different. A safe has a good passcode, and it opens if seven digits are entered and one of the digits matches the corresponding digit of the passcode. Is there a method of opening the safe box with an unknown passcode using less than 7 attempts?

2. $A,\ B,\ C,\ D,\ E$ and F are points in space such that AB is parallel to DE, BC is parallel to EF, CD is parallel to FA, but $AB \neq DE$. Prove that all six points lie in the same plane.

3. Are there positive integers $a,\ b,\ c$ and d such that $a^3 + b^3 + c^3 + d^3 = 100^{100}$?

4. A point is chosen on each side of a regular 2009-gon. Let S be the area of the 2009-gon with vertices at these points. For each of the chosen points, reflect it across the midpoint of its side. Prove that the 2009-gon with vertices at the images of these reflections also has area S.

5. A country has two capitals and several towns. Some of them are connected by roads. Some of the roads are toll roads where a fee is charged for driving along them. It is known that any route from the south capital to the north capital contains at least ten toll roads. Prove that all toll roads can be distributed among ten companies so that anybody driving from the south capital to the north capital must pay each of these companies.

Note: The problems are worth 4, 4, 4, 4 and 5 points respectively.

[1] Courtesy of Andy Liu.