

**International Mathematics
TOURNAMENT OF THE TOWNS**

Junior O-Level Paper

Fall 2009.¹

1. Is it possible to cut a square into nine squares and colour one of them white, three of them grey and five of them black, such that squares of the same colour have the same size and squares of different colours will have different sizes?
2. There are forty weights: 1, 2, \dots , 40 grams. Ten weights with even masses were put on the left pan of a balance. Ten weights with odd masses were put on the right pan of the balance. The left and the right pans are balanced. Prove that one pan contains two weights whose masses differ by exactly 20 grams.
3. A cardboard circular disk of radius 5 centimetres is placed on the table. While it is possible, Peter puts cardboard squares with side 5 centimetres outside the disk so that:
 - (1) one vertex of each square lies on the boundary of the disk;
 - (2) the squares do not overlap;
 - (3) each square has a common vertex with the preceding one.Find how many squares Peter can put on the table, and prove that the first and the last of them must also have a common vertex.
4. We only know that the password of a safe consists of 7 different digits. The safe will open if we enter 7 different digits, and one of them matches the corresponding digit of the password. Can we open this safe in less than 7 attempts?
5. A new website registered 2000 people. Each of them invited 1000 other registered people to be their friends. Two people are considered to be friends if and only if they have invited each other. What is the minimum number of pairs of friends on this website?

Note: The problems are worth 3, 4, 4, 5 and 5 points respectively.

¹Courtesy of Andy Liu