International Mathematics
TOURNAMENT OF THE TOWNS

Senior O-Level Paper **Spring 2008.**

1. There are ten cards with the number a on each, ten with b and ten with c, where a, b and c are distinct real numbers. For every five cards, it is possible to add another five cards so that the sum of the numbers on these ten cards is 0. Prove that one of a, b and c is 0.

2. Can it happen that the least common multiple of 1, 2, $\ldots$, n is 2008 times the least common multiple of 1, 2, $\ldots$, m for some positive integers m and n?

3. In triangle ABC, $\angle A = 90°$. M is the midpoint of BC and H is the foot of the altitude from A to BC. The line passing through M and perpendicular to AC meets the circumcircle of triangle AMC again at P. If BP intersects AH at K, prove that $AK = KH$.

4. No matter how two copies of a convex polygon are placed inside a square, they always have a common point. Prove that no matter how three copies of the same polygon are placed inside this square, they also have a common point.

5. We may permute the rows and the columns of the table below. How may different tables can we generate?

1	2	3	4	5	6	7
7	1	2	3	4	5	6
6	7	1	2	3	4	5
5	6	7	1	2	3	4
4	5	6	7	1	2	3
3	4	5	6	7	1	2
2	3	4	5	6	7	1

Note: The problems are worth 4, 5, 5, 5 and 6 points respectively.