International Mathematics
TOURNAMENT OF THE TOWNS

Junior O-Level Paper   **Spring 2008.**

1. In the convex hexagon $ABCDEF$, AB, BC and CD are respectively parallel to DE, EF and FA. If $AB = DE$, prove that $BC = EF$ and $CD = FA$.

2. There are ten congruent segments on a plane. Each point of intersection divides every segment passing through it in the ratio 3:4. Find the maximum number of points of intersection.

3. There are ten cards with the number a on each, ten with b and ten with c, where a, b and c are distinct real numbers. For every five cards, it is possible to add another five cards so that the sum of the numbers on these ten cards is 0. Prove that one of a, b and c is 0.

4. Find all positive integers n such that $(n + 1)!$ is divisible by $1! + 2! + \cdots + n!$.

5. Each cell of a 10×10 board is painted red, blue or white, with exactly twenty of them red. No two adjacent cells are painted in the same colour. A domino consists of two adjacent cells, and it is said to be good if one cell is blue and the other is white.

   (a) Prove that it is always possible to cut out 30 good dominoes from such a board.

   (b) Give an example of such a board from which it is possible to cut out 40 good dominoes.

   (c) Give an example of such a board from which it is not possible to cut out more than 30 good dominoes.

Note: The problems are worth 4, 5, 5, 5 and 6 points respectively.