

**International Mathematics
TOURNAMENT OF THE TOWNS**

Senior O-Level Paper¹

Spring 2007.

1. A 9×9 chessboard with the standard checkered pattern has white squares at its four corners. What is the least number of rooks that can be placed on this board so that all the white squares are attacked? (A rook also attacks the square it is on, in addition to every other square in the same row or column.)
2. The polynomial $x^3 + px^2 + qx + r$ has three roots in the interval $(0,2)$. Prove that $-2 < p + q + r < 0$.
3. B is a point on the line which is tangent to a circle at the point A . The line segment AB is rotated about the centre of the circle through some angle to the line segment $A'B'$. Prove that the line AA' passes through the midpoint of BB' .
4. A binary sequence is constructed as follows. If the sum of the digits of the positive integer k is even, the k -th term of the sequence is 0. Otherwise, it is 1. Prove that this sequence is not periodic.
5. A triangular pie has the same shape as its box, except that they are mirror images of each other. We wish to cut the pie in two pieces which can fit together in the box without turning either piece over. How can this be done if
 - (a) one angle of the triangle is obtuse and is twice as big as one of the acute angles;
 - (b) the angles of the triangle are 20° , 30° and 130° ?

Note: The problems are worth 3, 4, 4, 4 and 3+3 points respectively.

¹Courtesy of Professor Andy Liu