International Mathematics
TOURNAMENT OF THE TOWNS

O-Level Paper   **Spring 2006.**[2]

1   All vertices of a convex polyhedron with 100 edges are cut off by some planes. The planes do not intersect either inside or on the surface of the polyhedron. For this new polyhedron find

    a) [1] the number of vertices;

    b) [2] the number of edges.

2 [3] Do there exist functions $p(x)$ and $q(x)$, such that $p(x)$ is an even function while $p\big(q(x)\big)$ is an odd function (different from 0)?

3 [4] Let a be some positive number. Find the number of integer solutions x of inequality $100 < xa < 1000$ given that inequality $10 < xa < 100$ has exactly 5 integer solutions. Consider all possible cases.

4 [5] Quadrilateral $ABCD$ is a cyclic, $AB = AD$. Points M and N are chosen on sides BC and CD respectfully so that $\angle MAN = 1/2\,(\angle BAD)$. Prove that $MN = BM + ND$.

5   Pete has n^3 white cubes of the size $1 \times 1 \times 1$. He wants to construct a $n \times n \times n$ cube with all its faces being completely white. Find the minimal number of the faces of small cubes that Basil must paint (in black colour) in order to prevent Pete from fulfilling his task. Consider the cases:

    a) [3] $n = 3$;

    b) [3] $n = 1000$.

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[2]Your total score is based on the three problems for which you earn the most points. Points for each problem are shown in brackets [ ].