Juniors

(Grades up to 10)

International Mathematics
TOURNAMENT OF THE TOWNS

O-Level Paper **Fall 2005.**[1]

1 [**3**] In triangle ABC, points M_1, M_2 and M_3 are midpoints of sides AB, BC and AC, respectively, while points H_1, H_2 and H_3 are bases of altitudes drawn from C, A and B, respectively. Prove that one can construct a triangle from segments H_1M_2, H_2M_3 and H_3M_1.

2 [**3**] A number is written in each corner of the cube. On each step, each number is replaced with the average of three numbers in the three adjacent corners (all the numbers are replaced simultaneously). After ten such steps, every number returns to its initial value. Must all numbers have been originally equal?

3 [**4**] A segment of unit length is cut into eleven smaller segments, each with length of no more than a. For what values of a, can one guarantee that any three segments form a triangle?

4 [**4**] A chess piece moves as follows: it can jump 8 or 9 squares either vertically or horizontally. It is not allowed to visit the same square twice. At most, how many squares can this piece visit on a 15×15 board (it can start from any square)?

5 [**5**] Among 6 coins one is counterfeit (its weight differs from that real one and neither weights is known). Using scales that show the total weight of coins placed on the cup, find the counterfeit coin in 3 weighings.

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[1] Your total score is based on the three problems for which you earn the most points. Points for each problem are shown in brackets [ ].