International Mathematics
TOURNAMENT OF THE TOWNS

A-Level Paper   **Fall 2005.**[1]

1 [3] A palindrome is a positive integer which reads in the same way in both directions (for example, 1, 343 and 2002 are palindromes, while 2005 is not). Is it possible to find 2005 pairs in the form of $(n, n + 110)$ where both numbers are palindromes?

2 [5] The extensions of sides AB and CD of a convex quadrilateral $ABCD$ intersect at K. It is known that $AD = BC$. Let M and N be the midpoints of sides AB and CD. Prove that the triangle MNK is obtuse.

3 [6] Originally, every square of 8×8 chessboard contains a rook. One by one, rooks which attack an odd number of others are removed. Find the maximal number of rooks that can be removed. (A rook attacks another rook if they are on the same row or column and there are no other rooks between them.)

4 [6] Two ants crawl along the perimeter of a polygonal table, so that the distance between them is always 10 cm. Each side of the table is more than 1 meter long. At the initial moment both ants are on the same side of the table.

    (a) [2] Suppose that the table is a convex polygon. Is it always true that both ants can visit each point on the perimeter?

    (b) [4] Is it always true (this time without assumption of convexity) that each point on the perimeter can be visited by at least one ant?

5 [7] Find the largest positive integer N such that the equation $99x + 100y + 101z = N$ has an unique solution in the positive integers x, y, z.

6 [8] Karlsson-on-the-Roof has 1000 jars of jam. The jars are not necessarily identical; each contains no more than $\dfrac{1}{100}$-th of the total amount of the jam. Every morning, Karlsson chooses any 100 jars and eats the same amount of the jam from each of them. Prove that Karlsson can eat all the jam.

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[1] Your total score is based on the three problems for which you earn the most points. Points for each problem are shown in brackets [ ].