Seniors

(Grades 11 and up)

International Mathematics
TOURNAMENT OF THE TOWNS

O-Level Paper **Spring 2004.**

1 [4] Segments AB, BC and CD of the broken line $ABCD$ are equal and are tangent to a circle with centre at the point O. Prove that the point of contact of this circle with BC, the point O and the intersection point of AC and BD are collinear.

2 [4] A positive integer $a > 1$ is given (in decimal notation). We copy it twice and obtain a number $b = \overline{aa}$ which happened to be a multiple of a^2. Find all possible values of $\dfrac{b}{a^2}$.

3 [4] Perimeter of a convex quadrilateral is 2004 and one of its diagonals is 1001. Can another diagonal be 1? 2? 1001?

4 [5] Arithmetical progression $a_1, a_2, a_3, a_4, \ldots$ contains a_1^2, a_2^2 and a_3^2 at some positions. Prove that all terms of this progression are integers.

5 [5] Two 10-digit integers are called neighbours if they differ in exactly one digit (for example, integers 1234567890 and 1234507890 are neighbours). Find the maximal number of elements in the set of 10-digit integers with no two integers being neighbours.