International Mathematics
TOURNAMENT OF THE TOWNS

O-Level Paper **Fall 2004.**[1]

1 [3] Is it possible to arrange integers from 1 to 2004 in some order so that the sum of any 10 consecutive numbers is divisible by 10?

2 [4] A box contains red, green, blue, and white balls; 111 balls in total. It is known that among any 100 of them there are always balls of all 4 colors in mention.

Find the minimal number N such that among any N balls there are always balls of at least 3 different colors.

3 [4] A country consists of several cities; some of them are connected by Direct Express buses (each route connects two cities without intermediate stops).

Mr. Poor bought one ticket for every bus route while Mr. Rich bought n tickets for every bus route (a ticket allows a single one-way travel in either direction). Both Mr. Poor and Mr. Rich started from town A. Mr. Poor finished his travel in town B using up all his tickets without buying extra ones. Mr. Rich, after using some of his tickets, got stuck in town X: he cannot leave it without buying a new ticket. Prove that X is either A or B.

4 [5] A circle and a straight line with no common points are given. With compass and straightedge construct a square with two adjacent vertices on the circle and two other vertices on the line (it is known that such a square exists).

5 [5] Find the number of ways to decompose 2004 into a sum of positive integers (one or more) that all are "approximately equal".

Decompostions obtained from one another by permutations are not considered as different.

Two numbers are called *approximately equal* if their difference is at most 1.

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[1]Your total score is based on the three problems for which you earn the most points. Points for each problem are shown in brackets [ ].