Seniors
(Grades 11 and up)

International Mathematics
TOURNAMENT OF THE TOWNS

A-Level Paper   **Spring 2003.**

1 **[4]** A triangular pyramid $ABCD$ is given. Prove that $R/r > a/h$, where R is the radius of the circumscribed sphere, r is the radius of the inscribed sphere, a is the length of the longest edge, h is the length of the shortest altitude (from a vertex to the opposite face).

2 **[5]** $P(x)$ is a polynomial with real coefficients such that $P(a_1) = 0$, $P(a_{i+1}) = a_i$ $(i = 1, 2, \ldots)$ where $\{a_i\}_{i=1,2,\ldots}$ is an infinite sequence of distinct natural numbers. Determine the possible values of degree of $P(x)$.

3 **[5]** Can one cover a cube by three paper triangles (without overlapping)?

4 **[6]** A right $\triangle ABC$ with hypothenuse AB is inscribed in a circle. Let K be the midpoint of the arc BC not containing A, N the midpoint of side AC, and M a point of intersection of ray KN with the circle. Let E be a point of intersection of tangents to the circle at points A and C.

Prove that $\angle EMK = 90°$.

5 **[6]** Prior to the game John selects an integer greater than 100.

Then Mary calls out an integer d greater than 1. If John's integer is divisible by d, then Mary wins. Otherwise, John subtracts d from his number and the game continues (with the new number). Mary is not allowed to call out any number twice. When John's number becomes negative, Mary loses. Does Mary have a winning strategy?

6 **[7]** The signs "+" or "-" are placed in all cells of a 4×4 square table. It is allowed to change a sign of any cell altogether with signs of all its adjacent cells (i.e. cells having a common side with it). Find the number of different tables that could be obtained by iterating this procedure.

7 **[8]** A square is triangulated in such way that no three vertices are colinear. For every vertex (including vertices of the square) the number of sides issuing from it is counted. Can it happen that all these numbers are even?

Keep the problem set.
Visit: `http://www.math.toronto.edu/oz/turgor/`