Seniors
(Grades 11,12)

International Mathematics
TOURNAMENT OF THE TOWNS

O-Level Paper **Fall 2003.**

1 **[3]** For any integer $n+1, \ldots, 2n$ (n is a natural number) consider its greatest odd divisor. Prove that the sum of all these divisors equals n^2.

2 **[4]** What least possible number of unit squares (1×1) must be drawn in order to get a picture of 25×25-square divided into 625 of unit squares?

3 **[5]** A salesman and a customer altogether have 1999 rubles in coins and bills of 1, 5, 10, 50, 100, 500 , 1000 rubles. The customer has enough money to buy a Cat in the Bag which costs the integer number of rubles. Prove that the customer can buy the Cat and get the correct change.

4 Each side of 1×1 square is a hypothenuse of an exterior right triangle. Let A, B, C, D be the vertices of the right angles and O_1, O_2, O_3, O_4 be the centers of the incircles of these triangles. Prove that

a) [3] The area of quadrilateral $ABCD$ does not exceed 2;

b) [3] The area of quadrilateral $O_1O_2O_3O_4$ does not exceed 1.

6 **[5]** A paper tetrahedron is cut along some of so that it can be developed onto the plane. Could it happen that this development cannot be placed on the plane in one layer?