Juniors
(Grades up to 10)

International Mathematics
TOURNAMENT OF THE TOWNS

O-Level Paper **Fall 2003.**

1 [**3**] There is $3 \times 4 \times 5$ - box with its faces divided into 1×1 - squares. Is it possible to place numbers in these squares so that the sum of numbers in every stripe of squares (one square wide) circling the box, equals 120?

2 [**4**] In 7-gon $A_1A_2A_3A_4A_5A_6A_7$ diagonals A_1A_3, A_2A_4, A_3A_5, A_4A_6, A_5A_7, A_6A_1 and A_7A_2 are congruent to each other and diagonals A_1A_4, A_2A_5, A_3A_6, A_4A_7, A_5A_1, A_6A_2 and A_7A_3 are also congruent to each other. Is the polygon necessarily regular?

3 [**4**] For any integer $n+1, \ldots, 2n$ (n is a natural number) consider its greatest odd divisor. Prove that the sum of all these divisors equals n^2.

4 [**4**] There are N points on the plane; no three of them belong to the same straight line. Every pair of points is connected by a segment. Some of these segments are colored in red and the rest of them in blue. The red segments form a closed broken line without self-intersections (each red segment having only common endpoints with its two neighbors and no other common points with the other segments), and so do the blue segments. Find all possible values of N for which such a disposition of N points and such a choice of red and blue segments are possible.

5 [**5**] 25 checkers are placed on 25 leftmost squares of $1 \times N$ board. Checker can either move to the empty adjacent square to its right or jump over adjacent right checker to the next square if it is empty. Moves to the left are not allowed. Find minimal N such that all the checkers could be placed in the row of 25 successive squares but in the reverse order.