International Mathematics
TOURNAMENT OF THE TOWNS

Senior A-Level Paper **Spring 2002.**

1. In triangle ABC, $\tan A$, $\tan B$ and $\tan C$ are integers. Find their values.

2. Does there exist a point A on the graph of $y = x^3$ and a point B on the graph $y = x^3 + |x| + 1$ such that the distance between A and B does not exceed $\frac{1}{100}$?

3. In an increasing infinite sequence of positive integers, every term starting from the 2002-th term divides the sum of all preceding terms. Prove that every term starting from some term is equal to the sum of all preceding terms.

4. The spectators are seated in a row with no empty places. Each is in a seat which does not match the spectator's ticket. An usher can order two spectators in adjacent seats to trade places unless one of them is already seated correctly. Is it true that from any initial arrangement, the usher can place all the spectators in their correct seats?

5. Let AA_1, BB_1 and CC_1 be the altitudes of an acute triangle ABC. Let O_A, O_B and O_C be the respective incentres of triangles AB_1C_1, BA_1C_1 and CA_1B_1. Let T_A, T_B and T_C be the points of tangency of the incircle of ABC with sides BC, CA and AB respectively. Prove that $T_AO_CT_BO_AT_CO_B$ is an equilateral hexagon.

6. The 52 cards in a standard deck are placed in a 13×4 array. If every two adjacent cards, vertically or horizontally, have either the same suit or the same value, prove that all 13 cards of the same suit are in the same row.

7. Do there exist irrational numbers a and b such that $a > 1$, $b > 1$ and $\lfloor a^m \rfloor$ differs $\lfloor b^n \rfloor$ for any two positive integers m and n?

Note: The problems are worth 4, 4, 5, 5, 6, 7 and 8 points respectively.