International Mathematics
TOURNAMENT OF THE TOWNS

Junior O-Level Paper (Grades up to 10) **Spring 2002.**

1 [**4**] There are many $a \times b$-rectangular cardboard pieces (a and b are positive integers, and $a < b$). It is given that by putting such rectangles together (without overlapping) one can make 49×51-rectangle, and 99×101-rectangle. Can one uniquely determine values of a and b from these conditions?

2 [**5**] Can any triangle be cut into four convex figures: a triangle, a quadrilateral, a pentagon, and a hexagon?

3 [**5**] The last digit of the number $x^2 + xy + y^2$ is zero (where x and y are positive integers). Prove that two last digits of this number are zeroes.

4 [**5**] Quadrilateral $ABCD$ is circumscribed about some circle and K, L, M, N are points of tangency of sides AB, BC, CD and DA respectively, S is an intersection point of the segments KM and LN. It is known that the quadrilateral $SKBL$ is cyclic. Prove that the quadrilateral $SNDM$ is also cyclic.

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a) [**3**] There are 128 coins of two different weights, 64 of each. How can one always find two different coins by performing no more than 7 weightings on a regular balance?

b) [**3**] There are eight coins of two different weights, four of each. How can one always find two different coins by performing two weightings on a regular balance?