PROBLEMS OF TOURNAMENT OF TOWNS

Spring 2001, Level A, Senior (grades 11-OAC)

Problem 1 [3] Find at least one polynomial $P(x)$ of degree 2001 such that $P(x)+P(1-x)=1$ holds for all real numbers x.

Problem 2 [5] At the end of the school year it became clear that for any arbitrarily chosen group of no less than 5 students, 80% of the marks "F" received by this group were given to no more than 20% of the students in the group. Prove that at least 3/4 of all "F" marks were given to the same student.

Problem 3 [5] Let AH_A, BH_B and CH_C be the altitudes of triangle $\triangle ABC$. Prove that the triangle whose vertices are the intersection points of the altitudes of $\triangle AH_BH_C$, $\triangle BH_AH_C$ and $\triangle CH_AH_B$ is congruent to $\triangle H_AH_BH_C$.

Problem 4 [5] There are two matrices A and B of size $m \times n$ each filled only by "0"s and "1"s. It is given that along any row or column its elements do not decrease (from left to right and from top to bottom). It is also given that the numbers of "1"s in both matrices are equal and for any $k=1,\dots,m$ the sum of the elements in the top k rows of the matrix A is no less than that of the matrix B. Prove for any $l=1,\dots,n$ the sum of the elements in left l columns of the matrix A is no greater than that of the matrix B.

Problem 5 In a chess tournament, every participant played with each other exactly once, receiving 1 point for a win, 1/2 for a draw and 0 for a loss.

(a) [4] Is it possible that for every player P, the sum of points of the players who were beaten by P is greater than the sum of points of the players who beat P?

(b) [4] Is it possible that for every player P, the first sum is less than the second one?

Problem 6 [8] Prove that there exist 2001 convex polyhedra such that any three of them do not have any common points but any two of them touch each other (i.e., have at least one common boundary point but no common inner points).

Problem 7 Several boxes are arranged in a circle. Each box may be empty or may contain one or several chips. A move consists of taking all the chips from some box and distributing them one by one into subsequent boxes clockwise starting from the next box in the clockwise direction.

(a) [4] Suppose that on each move (except for the first one) one must take the chips from the box where the last chip was placed on the previous move. Prove that after several moves the initial distribution of the chips among the boxes will reappear.

(b) [4] Now, suppose that in each move one can take the chips from any box. Is it true that for every initial distribution of the chips you can get any possible distribution?