Junior O-Level Paper   **Fall 2001.**

1. In the quadrilateral $ABCD$, AD is parallel to BC. K is a point on AB. Draw the line through A parallel to KC and the line through B parallel to KD. Prove that these two lines intersect at some point on CD.

2. Clara computed the product of the first n positive integers and Valerie computed the product of the first m even positive integers, where $m \geq 2$. They got the same answer. Prove that one of them had made a mistake.

3. Kolya is told that two of his four coins are fake. He knows that all real coins have the same weight, all fake coins have the same weight, and the weight of a real coin is greater than that of a fake coin. Can Kolya decide whether he indeed has exactly two fake coins by using a balance twice?

4. On an east-west shipping lane are ten ships sailing individually. The first five from the west are sailing eastwards while the other five ships are sailing westwards. They sail at the same constant speed at all times. Whenever two ships meets, each turns around and sails in the opposite direction. When all ships have returned to port, how many meetings of two ships have taken place?

5. On the plane is a set of at least four points. If any one point from this set is removed, the resulting set has an axis of symmetry. Is it necessarily true that the whole set also has an axis of symmetry?

Note: Each problem is worth 4 points.