

MAT 224H1 Linear Algebra II: Fall 2018

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This syllabus is very long, but it contains lots of important information, including most of the types of questions you're likely to have throughout the semester. Not reading it, and asking the instructor a question easily located here, is bad etiquette. The above table of contents is "clickable" for easy navigation.

1 Course Website

All important information is regularly maintained on the course website

<http://www.math.toronto.edu/nhoell/MAT224>

Please bookmark the page for your convenience and check in regularly. You are expected to **check your email regularly** throughout the semester as you will receive exam and other course-related announcements sent to your inbox throughout the semester. Important **announcements** will be posted on the course website as well. If you ask an instructor questions whose answer is easily found in the main course page you may find your Instructor staring at you quizzically. If you email your Instructor asking a question whose answer is easily found on the course website, you should expect no response for what are, I hope, obvious reasons.

2 Content of this Course

The goal of this course, naively, is to answer the question "What's a vector?" If you ask 10 mathematicians that question you will probably get close to 10 different answers. If you ask me, I'd say "a vector is anything which acts like a vector". We will spend 3 months trying to unpack that unsatisfying (and unhelpful) answer to this basic question. Vectors have tremendous applications across a variety of disciplines including *physics, economics, chemistry, statistics, mathematics, operations research, artificial intelligence, computer science, imaging, neuroscience, machine learning*, and other areas of science and, as such, it's very important that we have a deep understanding of what they are like.

MAT224 is considered an introduction to linear algebra at a more advanced mathematical level than MAT223; we will greatly expand on what you have seen in MAT223, abstracting out the fundamental concepts. The material may feel quite abstract and removed from direct applications at times but we will try to stay as grounded as possible and highlight the importance and utility of all the general techniques we present. Mastery of this material is the first step on the long journey towards a proper scientific training, as it is in many ways the de facto *lingua franca* in the sciences.

2.1 Prerequisites

The University prerequisites for the course are given in the following subsection. But those are purely administrative (i.e. they are just the sufficient conditions to be able to register and say nothing about your chances of success). To do well in this class requires **having a solid acquaintance with the material in MAT223**. If you scored under a 75% in MAT223 this course may prove to be quite demanding. We will be **focussed much more on proofs than in prior courses**, and the material we cover is more abstract. For this reason, you should review the content of MAT223, in particular the material on subspaces, rank, dimension and diagonalization. If it has been awhile since you took MAT223 then reviewing this material, prior to the start of MAT224, is even more important. You should be competent at writing readable and coherent (and correct!) proofs.

Technical Details

Make sure you are aware of the following University details regarding this course.

- Prerequisites: MAT221H1(80%)/ MAT223H1/ MAT240H1
- Exclusion: MAT247H1
- Distribution Requirement Status: Science
- Breadth Requirement: The Physical and Mathematical Universes (5)
- Program Area Section: Mathematics

3 Lecture Sections

There are two instructors for the course as listed below.

LECO101: T 11:00–13:00 & R 11:00–12:00 in MP202

Instructor & Coordinator: Professor Nicholas Hoell

Email: nicholashoell@gmail.com

LECO401: T 12:00–13:00, W 14:00–15:00 & R 12:00–13:00 in MP103

Instructor: Professor Sean Uppal

Email: uppal@math.utoronto.ca

LECO201: T 18:00–21:00 in SF1105

Instructor & Coordinator: Professor Nicholas Hoell

Email: nicholashoell@gmail.com

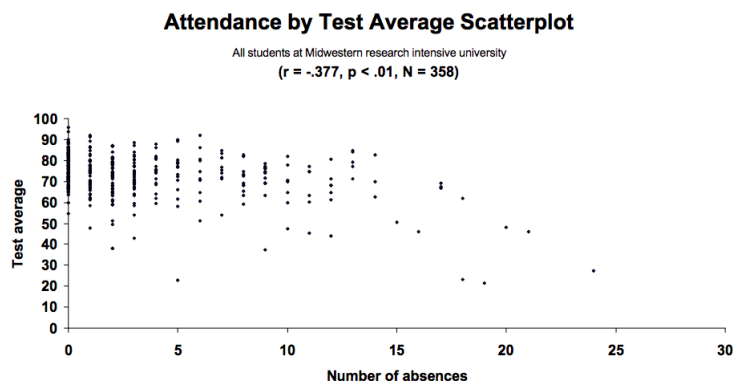
3.1 Why Attend Lectures?

None of the instructors take attendance, although we all **strongly encourage attendance** as it is the best way to stay on top of material throughout the term as well as to get viewpoints and idiosyncrasies particular to the instructors (who, after all, are writing the exams you'll be taking!). Here are my thoughts on why you should attend.

1. Attendance of **every single lecture** is well correlated with success on our assessments. See, for example, the article

<http://communication.utsa.edu/leblanc/articles/art31.pdf>

which studied the effects of attendance on performance (using techniques based on linear algebra!) at four American universities. If you don't want to read the article, you should be aware that it contains several scatterplots which look like this



There are a number of issues about the interpretability and applicability of plots like the above. For example, this is a different university, with different instructors, and the course itself is different. As well, our test averages may be quite different from those in the paper. So, take the above as a very rough indicator of the expected negative effect of absences on assessment.

2. You (or your parents or your scholarship organization or...) pay for the lectures. Imagine someone who routinely paid for a meal and a seat at a(n expensive) restaurant, but ate most of their meals at home.
3. There are things you simply cannot get from textbooks that you can get from lectures. Your instructors are mathematicians. They all know, and regularly use, ideas from this course in their professional lives. They know how mathematicians think, and they know well the "folk history" and "folk theorems" of the subject. There are aspects of the *culture of mathematics* which haven't made it into books and exist only in the oral traditions. And these offer some of the deepest insight into the material. Not attending lecture means you're not utilizing possibly the greatest resource that a University like this one has - namely, access to experienced practitioners.

3.2 Lecture Behaviour

Here are some additional thoughtlets on lecture behaviour and participation.

1. Don't feel shy about raising your hand and asking for clarification on points of confusion. Whatever your confusion, there are no doubt others in the room who share it. Moreover, **the instructors of MAT224 welcome and encourage your active participation**. It helps *us* learn about what students may be struggling with.
2. There is a big difference between being merely *physically present* and being *intellectually present*. Being physically present is no harder than finding a seat. Being intellectually present requires paying attention. In particular texting, being on your laptop, having earbuds in, chatting, or generally showing obvious signs of disengagement not only hinders your ability to learn but they are things your instructors will point out and ask you to stop doing.
3. If you miss a lecture due to illness or other understandable, unavoidable, reasons, feel free to ask your instructor after the next lecture you attend about topics you may have missed. In addition you will find it helpful to make friends over the term with whom you can share notes. And, failing that, this course will be having a Piazza forum accompanying it. Therefore, simply sending an email to your instructor along the lines of "I was sick yesterday what did you cover?" however will, for what I hope are obvious reasons, go unanswered.

4 Tutorials

There are tutorial sections for this course and you must be registered to one. TBD

5 Textbook

5.1 Required Texts

Our required text this semester is *A Course in Linear Algebra* by David Damiano, and John Little, published by Dover, ISBN-13: 978-0-486-46908-9. It is a clean, well-written softcover well suited for this course. If **you decide to use an older edition** of the book, that's fine but you **do so at your own risk** because the suggested problems may not match those which are in the Dover edition. It is **the students' responsibility alone** to make sure you are working through analogous problems if in fact you decide to use an older edition. **If you ask your instructor about using other editions, they will look at you quizzically and point you to this syllabus.**

5.2 Optional Text

You may also wish to consult the text *Linear Algebra* by Friedberg, Insel & Spence, 4th ed. ISBN: 978-0-130-08451-4. This book is often used for the specialist linear algebra course MAT240/MAT247. It's a well written text, with great problems. It happens to be too advanced for this course but for those of you looking to delve deeper, this is a great text to read.

Another excellent book is *Linear Algebra Done Right* by Axler, 3rd ed. ISBN: 978-3-319-11079-0 published by Springer. This book is a bit more advanced than our course but gives a good treatment of the material we'll be covering.

5.3 Other Materials

In addition to the textbook, we will occasionally post additional problems and references on the course website as the semester progresses. Please don't limit yourself solely to the main texts!

Some students in the past found the following videos to be helpful.

https://www.youtube.com/playlist?list=PLZHQ0bOWTQDPD3MizzM2xVFitgF8hE_ab

5.4 A Word on Materials

Be aware that course materials are provided for the use of enrolled students only and that registered students are not allowed to post, share, or sell course materials without the instructor's permission.

6 Piazza Forum

We will be having class discussions on the Piazza discussion forum.

<https://piazza.com/utoronto.ca/fall2018/mat224h1f>

You should know that this is a place where you can go to (anonymously, if you wish) ask questions and get feedback from lots of other students who are in the class. You'll get the most benefit by trying to answer questions posed by other students and, for the most part, the TAs and Instructors who frequent the site do not actively answer questions but are there to ensure that answers floating around are, in fact, correct so that misinformation doesn't spread. Students benefited greatly from the Piazza forums in the past and I suggest enrolling as soon as possible.

7 Homework

You should do **all** suggested problems listed on the course website. Nothing is to be handed in. Trying to do well in MAT224 without doing the homework is equivalent to trying to play a Chopin etude purely by watching others play the piece. To do well in mathematics you *actually have to do mathematics*.

8 Tutorials

Part of your grade in this course is based on tutorials. **Tutorials begin on the week of September 17th**. Your grade in tutorials is based on attendance. To receive attendance marks requires being in *full attendance* for the complete tutorial. In principle, these should be the easiest marks you'll earn in University. No medical notes are needed, nor will any be accepted, for missed tutorials as the lowest two scores will be dropped from your score. **You are expected to bring your student identification (T-card), paper and a pen/pencil to tutorials**

9 Grading

Grades will be based according to some quizzes throughout the term, two midterm exams and a final exam. Your final grade in the course will be determined by the following

- *Tutorials: 5%; the lowest two scores will be dropped*
- *Midterms: 25% for the highest, 20% for the lowest*
- *Final Exam: 50%*

10 Tests

There are two midterms during the semester. They are held on

1. Midterm 1: Friday, October 5th, 3:10-4:00PM
2. Midterm 2: Friday, November 2nd, 3:10-4:00PM

Note on Exams: No electronic devices/aids will be allowed during the exams. It is the students' responsibility to ensure that the allotted exam time is available.

Arrangements may be made for a test on the **same day but an alternate time** for students who have a regularly scheduled University of Toronto class (or laboratory) scheduled at that time. This **requires presenting evidence of your enrollment** in such a course (e.g. your schedule from ACORN/ROSI) **prior** to the week of the midterm. If you are such a student you **you'll be asked to provide proof of such a conflict as the exam approaches in an email announcement**. Without proof of course conflicts, and without proper notification, you **will not be allowed to take the midterm at an alternative time**.

10.1 Multiple Tests

It should be mentioned that *having multiple tests on the same day* does **not** count as a course conflict. If you happen to have multiple tests on the day of one of ours but there is no overlap in the hours the test is held **this does not count** as a course conflict and you are expected to write our midterm.

10.2 Missed Tests

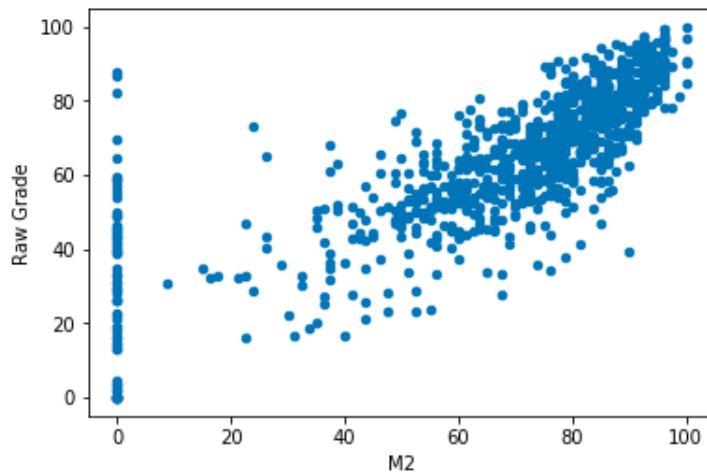
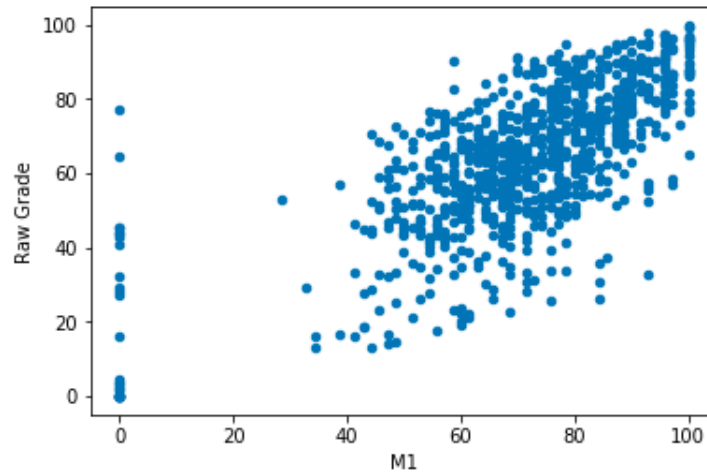
There are no makeup tests. A student presenting proof of a **valid** reason for missing a test (see the section on Missed Term Tests in the Rules and Regulations section of the Faculty of Arts and Science 2016-2017 Calendar as well as the following section of this syllabus) will have their grading scheme adjusted to the following

- *Tutorials: 5%; the lowest two score will be dropped*
- *Midterm: 30% for the midterm taken*
- *Final Exam: 65%*

In the unlikely event that there are two **VALID** medical excuses, this procedure will not be followed. Students in this circumstance will have their mark depend 95% on the final exam and 5% on tutorials. Almost **every student who has been in this situation has failed the course** due to a very low mark on the final exam. It is **STRONGLY ADVISED** that you write all 2 term tests. Missing both tests and only taking the final exam is a **near guarantee of failing the course**.

10.3 Implications of Missing Tests

Below, you will find scatterplots showing grades on the midterms 1 and 2 plotted versus final course score in MAT223 from Spring of 2018. The vertical line of dots above 0 on the x axis indicates people who were absent during the given midterm (the score was, for excused absences, not counted as a zero but re-weighted according to a similar scheme as outlined in the previous section). You'll notice that, in many instances, those falling in that category did not pass the course. It is not advisable to miss a midterm **unless truly necessary for health reasons**. Missing a term test in order to "buy time" to help prepare for the final exam is not a winning strategy.



10.4 Medical Notes

In the case of a legitimate medical issue **medical notes will be accepted ONLY from MDs with a valid CPSO number**. You must present your section Instructor with a University of Toronto Verification of Student Illness or Injury form available at <http://www.illnessverification.utoronto.ca/index.php>.

Some important remarks about these notes.

- These forms **must be submitted to your course instructor within 3 business days of the missed test** for the absence to not be penalized. Failure to submit proper, valid and timely documentation will result in a grade of 0 on your missed test.

- The form must have all required fields filled properly and *legibly*.
- The form must be original.
- The form is only considered valid if **completed by a qualified medical doctor - not an acupuncturist, chiropractor, naturopath or other health care professional.**
- Upon submission of the documentation review of the medical note will be done before it is accepted as valid. The review **may include following up with your doctor, your college registrar, or other departmental advisors.**

Presenting a false medical excuse is a severe offence and will be dealt with through the Office of the Dean of the Faculty of Arts and Science.

10.5 Athletic Absences

If you are a member of a University of Toronto sports team which has an event scheduled on the date of one of our tests and you wish to not miss your event then you **must get a letter on University letterhead** from your coach in order for this to count as an excused absence. The same grading policy towards excused medical absences applies in this instance. The only difference is that **you must have the letter sent to us prior to the week of the midterm you plan on missing.**

11 Contact

11.1 Email Etiquette

It is University policy that **instructors need only reply to emails sent from University email accounts.** Acceptable emails are of the form student@utoronto.ca, topstudent@math.toronto.edu, etc. We will not likely ever reply to a non-University email address (those from addresses like, say, studentwhoseemaildidntgetreturned@hotmail.com or studentwhodidntreadthesyllabus@gmail.com). Any email should have the words "MAT223" somewhere in the subject line.

As for email etiquette, sending technical mathematics questions to your instructor is fine if they are **very short** and worded *very* precisely. Save longer questions for the beginning Q&A part of the following lecture, or ask during office hours, or during your tutorials, or at the Math Aid Centre. We also wish for you to use the online Piazza forum for discussing questions with each other (which we monitor to avoid misinformation spreading).

You should **not address us** as "Hey", "Yo", or other highly informal salutation in an email. If you do you probably won't get a reply. Keep in mind we get a high volume of emails each semester and it may take time before you hear back from us (if at all). Your chances of having your email read and being replied to increase *dramatically* if you can follow the instructions above.

11.2 Whom do I contact?

1. For any math-related questions ask any instructor (see above), ask on the Piazza forum, or ask your TA in tutorials, or ask in the MathAid Centre (see below)
2. For enrolment changes contact your college registrar
3. If you have a regularly scheduled University of Toronto course which conflicts with a test date, you'll be asked to contact the coordinator via email once the exam is nearing, no need to pre-empt this with emails prior to being requested.

4. For questions about which math course is right for you, contact the Undergraduate Administrator Donna Birch at dbirch@math.utoronto.ca, or in BA6291 or via 416-978-5082.
5. If you require accommodations for a disability, or have any accessibility concerns about the course, the classroom, or course materials, please contact Accessibility Services: <http://www.accessibility.utoronto.ca/index.htm>
6. If you have a personal situation and are concerned about how it may affect your academic performance, please contact your college registrar.

There are TAs for MAT224 regularly assigned to the MathAid center which will be open for help during the academic year at hours maintained on the course website. The MathAid Centre is on the main floor of the Physical Geography Building located on 45 St. George St.

Quercus is used by instructors of MAT224 for sending out email to the class or when entering marks on tests and tutorials. For any important course information and announcements you should look to the course web site listed at the front of this syllabus.

12 Academic Integrity (Important) ---

Cheating (including plagiarism) is very serious and, consequently, will be taken **very** seriously. Cheating can result in failure **or worse**. Don't do it! I caution you, the instructors of MAT224 are *extremely* diligent in pushing for the **maximum possible penalties** for those found cheating. Any collusion or fabrication during or after test/quiz situations will be vigorously pursued. This includes talking (or making other extraneous noises of any kind) during a test. We don't tolerate any kind of chatter during tests.

One other thing. There are students for whom the statement "The test is now over, please put your pens and pencils down while we collect the tests" seems to not entirely register. We consider egregious dismissals of our requests to stop writing to be a form of academic integrity violations which we enforce with the same stringency as talking during a test. **It's not worth the risk!** Every semester there are students who don't heed this warning, and every semester this situation is dealt with through administrative channels that have serious consequences for the student.

13 Important Dates ---

The following are a list of dates you may wish to know and whom to contact in special cases.

1. September 19: Last day to add the course.
2. October 5: Midterm #1
3. November 2: Midterm #2
4. November 5: Last day to drop without academic consequence. Also, last day to add or remove a CR/NCR option.
5. November 5-9: Reading Week
6. December 5: Last day of class. Also last day to request a late withdrawal from the college registrar's office.

14 External Resources

14.1 Accessibility Accommodations

The University of Toronto is committed to accessibility. As such, if you require accommodations for a disability, or have any other accessibility concerns about the course, the classroom or course materials, please contact Accessibility Services, <http://www.studentlife.utoronto.ca/> as soon as possible.

14.2 Writing and English Language Instruction

For information on campus writing centres and writing courses, please visit <http://www.writing.utoronto.ca/>. FREE English language instruction with the ELL Program will start in Winter 2016. The Communication Cafe offers drop-in discussions, presentations, and debates, along with learning about Canadian culture - no registration necessary. Sessions are facilitated by writing centre instructors. For more information about the English Learning Language (ELL) program, please visit <http://www.artsci.utoronto.ca/current/advising/ell>.

14.3 Other Resources

As well, you may wish to visit any of the following.

- Student Life Programs and Services: <http://www.studentlife.utoronto.ca>
 - Academic Success Centre: <http://www.studentlife.utoronto.ca/asc>
 - Health and Wellness Centre: <http://www.studentlife.utoronto.ca/hwc>
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15 The Menu

We will keep, *very roughly*, to the following schedule. The **schedule is not rigid** and may vary depending on which section you are enrolled in. You should consult the course webpage for more accurate information as the course moves on. The suggested problems will take you considerable time, but staying on top of them is the only way to master the material. We strongly advise you to do all of them, and to read the corresponding sections of the textbook. This, together with attendance of lectures and tutorials, may easily take more than 15 hours per week.

- Week 0
 - **Topics:** Sets, Logic and Proofs. Students should review this material on their own prior to the beginning of class.
 - **Suggested Problems:** Appendix: All problems.
- Week 1
 - **Topics:** Spaces, subspaces, and combinations.
 - **Suggested Problems:** Section 1.1: 1-12, Section 1.2: 1-16 & Section 1.3: 1-12
- Week 2
 - **Topics:** Combinations continued. Independence. Bases and Dimension.
 - **Suggested Problems:** Section 1.3: 1-12, Section 1.4: 1-12 & Section 1.6: 1-16. (Students should be familiar with Section 1.5 from MAT223).
- Week 3

- **Tutorials Begin**
- **Topics:** Linear transformations.
- **Suggested Problems:** Section 2.1: 1–15. Section 2.2: 1–15
- Week 4
- **Topics:** Kernel and image. Rank-nullity theorem.
- **Suggested Problems:** Section 2.3: 1–14 & Section 2.4: 1–12.
- Week 5
- **Midterm 1** on Friday, October 5
- **Topics:** Compositions and inverses.
- **Suggested Problems:** Section 2.5:1–19 & Section 2.6: 1–17
- Week 6
- **Topics:** Change of Basis. Similarity. Determinants Overview.
- **Suggested Problems:** Section 2.7: 1–9. Section 3.1: 1–12. Section 3.2: 1–16. Section 3.3: 1–12.
- Week 7
- **Topics:** Eigenproblems and Diagonalizability. Simultaneous diagonalizability and commutators.
- **Suggested Problems:** Section 4.1: 1–18 & Section 4.2: 1–15.
- Week 8
- **Midterm 2** on Friday, November 2
- **Topics:** Gram-Schmidt procedure. Symmetric matrices.
- **Suggested Problems:** Section 4.3: 1–17 (material not covered in lecture, students should do this on their own). Section 4.4: 1–19 & Section 4.5: 1–10
- Week 9
- **Topics:** The spectral theorem. Singular value decomposition. Complex numbers.
- **Suggested Problems:** Section 4.6: 1–17. Section 5.1: 1–15
- Week 10
- **Topics:** Complex vector spaces. Triangularity and Nilpotency.
- **Suggested Problems:** Section 5.2: 1–15, Section 5.3: 1–15, Section 6.1: 1–15, & Section 6.2: 1–16.
- Week 11
- **Topics:** Jordan canonical form. Time permitting - Pauli matrices.
- **Suggested Problems:** Section 6.3: 1–11, Section 6.4:1–5
- Week 12
- **Topics:** Buffer

In the above, section numbers refer to the Dover edition of Damiano and Little's text.