Tutorial 5201 (Thursday, 5-6), Quiz 1, rubric.

1. Use the Cauchy-Riemann equations to determine whether the following function is analytic on the complex plane:

$$\cosh(x^2 - y^2)\cos(2xy) + i\sinh(x^2 - y^2)\sin(2xy)$$

Solution: Let $P = \cosh(x^2 - y^2)\cos(2xy)$ and $Q = \sinh(x^2 - y^2)\sin(2xy)$ be the real and imaginary parts of the function, respectively. Then we have

$$\frac{\partial P}{\partial x} = 2x\sinh(x^2 - y^2)\cos(2xy) - 2y\cosh(x^2 - y^2)\sin(2xy),$$
$$\frac{\partial P}{\partial y} = -2y\sinh(x^2 - y^2)\cos(2xy) - 2x\cosh(x^2 - y^2)\sin(2xy),$$
$$\frac{\partial Q}{\partial x} = 2x\cosh(x^2 - y^2)\sin(2xy) + 2y\sinh(x^2 - y^2)\cos(2xy),$$
$$\frac{\partial Q}{\partial y} = -2y\cosh(x^2 - y^2)\sin(2xy) + 2x\sinh(x^2 - y^2)\cos(2xy),$$

from which we see that

$$\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y}$$

and

$$\frac{\partial P}{\partial y} = -\frac{\partial Q}{\partial x},$$

so that the Cauchy-Riemann equations are satisfied and the function is therefore analytic as the partial derivatives of P and Q are continuous.

Marking: 1 mark for each partial derivative, 1 mark for each of the Cauchy-Riemann equations, for 6 marks total. Mentioning continuity is *not* required since the problem was ambiguous as to whether that was needed or not.

2. Use the Cauchy-Riemann equations to determine whether the following function is analytic on the complex plane:

$$-\sinh(x^2 - y^2)\sin(2xy) + i\cosh(x^2 - y^2)\cos(2xy)$$

Solution: Let $P = -\sinh(x^2 - y^2)\sin(2xy)$ and $Q = \cosh(x^2 - y^2)\cos(2xy)$ be the real and imaginary parts of the function, respectively. Then we have

$$\frac{\partial P}{\partial x} = -2x\cosh(x^2 - y^2)\sin(2xy) - 2y\sinh(x^2 - y^2)\cos(2xy),$$
$$\frac{\partial P}{\partial y} = 2y\cosh(x^2 - y^2)\sin(2xy) - 2x\sinh(x^2 - y^2)\cos(2xy),$$
$$\frac{\partial Q}{\partial x} = 2x\sinh(x^2 - y^2)\cos(2xy) - 2y\cosh(x^2 - y^2)\sin(2xy),$$
$$\frac{\partial Q}{\partial y} = -2y\sinh(x^2 - y^2)\cos(2xy) - 2x\cosh(x^2 - y^2)\sin(2xy),$$

from which we see that

$$\frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y}$$

and

$$\frac{\partial P}{\partial y} = -\frac{\partial Q}{\partial x},$$

so that the Cauchy-Riemann equations are satisfied and the function is therefore analytic as the partial derivatives of P and Q are continuous.

Marking: 1 mark for each partial derivative, 1 mark for each of the Cauchy-Riemann equations, for 6 marks total. Mentioning continuity is *not* required since the problem was ambiguous as to whether that was needed or not.