MAT334, COMPLEX VARIABLES, SUMMER 2020. PROBLEMS FOR JUNE 8 – 12

Due Tuesday, June 16, at 3:30 PM EDT.

1. [9 marks] Let

$$f(x+iy) = \cos x \cosh y - i \sin x \sinh y.$$

By choosing and parameterising an appropriate (potentially only piecewise-smooth) curve, determine the function

$$F(z) = \int_0^z f(z')\, dz'.$$

Explain why the result does not depend on your choice of curve.

2. [9 marks] Using the Cauchy integral formula, evaluate the following integrals:

$$\int_\gamma \frac{\cos z}{z}\, dz, \qquad \gamma \text{ the square with sidelength 2 centred at the origin, oriented counterclockwise.}$$

$$\int_\gamma \frac{1}{(z-z_0)^2}\, dz, \qquad \gamma \text{ any simple closed curve containing the point } z_0\text{, in any orientation.}$$

$$\int_\gamma \frac{e^z}{z}\, dz, \qquad \gamma \text{ the unit circle, oriented } \textit{clockwise.}$$

3. [6 marks] Let γ denote the unit circle, oriented counterclockwise, and let $z^{1/2}$ denote any branch of the square root function (be sure to clearly indicate which one you are using!). By direct computation, evaluate the integral

$$\int_\gamma \frac{z^{1/2}}{z}\, dz,$$

where we can evaluate the integral since the function is defined and bounded everywhere except at a single point on the curve (alternatively, you can view the above integral as a limit of an open segment of the circle as the two endpoints come towards the branch cut). Does your result contradict the Cauchy integral theorem or formula? Why or why not?