MAT334, COMPLEX VARIABLES, SUMMER 2020. PROBLEMS FOR JUNE 1 – 5

Due Tuesday, June 9, at 3:30 PM EDT.

1. [6 marks] Without doing any differentiation, explain why the following functions are harmonic on the indicated regions:

$$\cos(\cos x \cosh y)\cosh(\sin x \sinh y)\,, \qquad \text{everywhere on the plane.}$$

$$\frac{1}{2}\log\left(\sin^2 x \cosh^2 y + \cos^2 x \sinh^2 y\right) \qquad \text{on the set } \{(x,y)|x,y>0\}.$$

2. [18 marks] Evaluate the following integrals:

$$\int_\gamma \frac{1}{z}\,dz, \qquad \text{where } \gamma \text{ represents the unit circle, traversed counterclockwise.}$$

$$\int_\gamma \frac{1}{z}\,dz, \qquad \text{where } \gamma \text{ represents the circle of radius one and centre } 2i, \text{ traversed counterclockwise.}$$

$$\int_\gamma \frac{1}{z^2}\,dz, \qquad \text{where } \gamma \text{ represents any circle centred at the origin.}$$

Do any of these results contradict the result we derived in class about integrals of analytic functions over closed curves? Do any of them add to that result? Why or why not?