Mat 240F - Final exam information

Location and date: Please consult the December examination schedule. (The time allowed for writing the exam is 3 hours.)

NO AIDS ALLOWED - you cannot use calculators, notes, text, or a formula sheet. Exam booklets will be provided: please bring your own pen or pencil.

Material for test: Material related to questions 3–6 from Problem Set 2, and questions from Problem Sets 3–8. (Note that there is some overlap with the term test material, since questions 3–6 of the term test are based on material from Problem Sets 2–4.). Most of the questions on the final exam will involve linear transformations. Note that results and techniques from Sections 1.3–1.6 are often used in the solution of questions about linear transformations.

Using theorems proved in class, in the text, or in problem sets: When using a theorem in solving a problem, you don't need to remember the number of the theorem or the section of the text in which it appears. You should state the fact and say it was proved in class, in the text, or on a problem set.

Review questions Problem set questions (starting with question 3 from Problem Set 2), plus the following questions:

> Section 1.3: 8, 10, 11–14; Section 1.4: 14; Section 1.5: 1–3, 13, 19; Section 1.6: 3, 4, 11, 14, 16, 26; Section 2.1: 1, 2, 9–14, 16–21; Section 2.2: 1–7, 10, 13; Section 2.3: 1–4, 9, 11, 12; Section 2.4: 1–7, 9–12, 14–18; Section 2.5: 4, 5; Section 3.1: 3, 5, 12; Section 3.2: 1, 2, 4–8, 11, 14, 17; Section 3.4: 2; Section 4.2: 5–22, 26; Section 4.3: 9, 11, 12, 17; Section 4.4: 3, 4; Section 5.1: 1–4, 8, 15, 17, 20, 22a), 23; Section 5.2: 1a)–g), 2, 3, 8, 10, 12, 13.

1. In each case below, determine whether or not the subset W of the vector space V is a subspace of V. If W is a subspace of V, prove it. If W is not a subspace of V, demonstrate how one of the properties of subspace fails to hold.
   a) Let V be a 3-dimensional vector space over $\mathbb{R}$ and let β be an ordered basis of V. If $x \in V$, let $[x]_\beta$ be the coordinate vector of x relative to the ordered basis β. Let
   $$W = \left\{ x \in V \mid [x]_\beta \in \operatorname{span}\left\{ \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \right\} \right\}.$$
   b) Let $V = \mathcal{L}(P_3(\mathbb{C}))$ and let
   $$W = \{ T \in V \mid T(x^3)T(x) = T(1) \}.$$
2. Let $V = M_{2\times 2}(\mathbb{R})$.
   a) Find an ordered basis β for V having the property that every matrix A that belongs to the basis β satisfies $\operatorname{trace}(A) \neq 0$. (Here, $\operatorname{trace}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = a + d$.)
   b) Let γ be a basis of V. Prove that there exists at least one $A \in \gamma$ for which $\operatorname{trace}(A) \neq 0$.
3. Find a basis for each vector space V below.
   a) Let V_0 be a vector space of dimension 4. Let $\beta = \{ x_1, x_2, x_3, x_4 \}$ be an ordered basis for V_0. Let
   $$V = \{ T \in \mathcal{L}(V_0) \mid T(x_1) + T(x_2) = T(x_4) \}.$$
   b) Let V_0 be as in part a). Let
   $$V = \{ T \in \mathcal{L}(V_0) \mid R(T) \subset \operatorname{span}(\{ x_1, x_2 \}) \}.$$