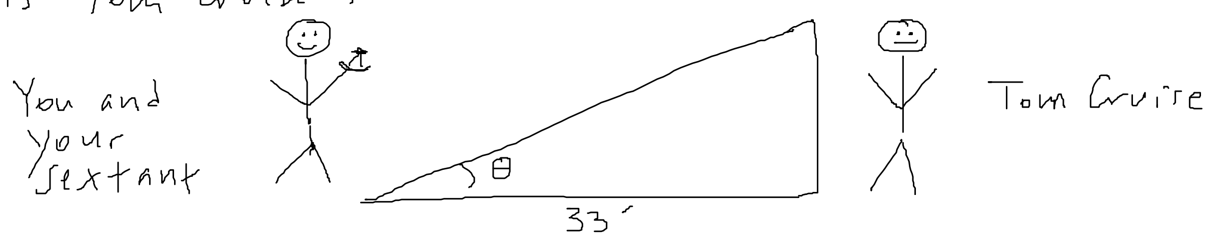
Mat 135, Sept 10 2004

In the next four lectures, we will cover introductory trigonometry and will review exp and ln. Please read Appendix D and Chapter 1 carefully and do all the recommended problems. If you have any questions, please see me or go to the math aid center.

You're walking down St George street and see Tom Cruise on the other side of the street. "How tall is he?!?" you ask yourself. You pull out your sextant and measure the angle $\theta = 9.6°$. If Tom Cruise is 33' distant from you, how tall is Tom Cruise?



How would your guess at Tom Cruise's height be affected if you had a 1 foot error in the distance between the two of you?

How would your guess at his height be affected if you were off by 1/2 a degree?

What to do? Use trig!

Before dealing with Tom Cruise, let's do some background work.

We will not use degrees in this course. Instead, we use radians. A complete revolution contains 360 degrees and contains 2π radians.

To convert from degrees to radians, multiply by $\frac{2\pi}{360} \frac{\text{radians}}{\text{degrees}}$. To convert from radians to degrees, multiply by $\frac{360}{2\pi} \frac{\text{degrees}}{\text{radians}}$

for example

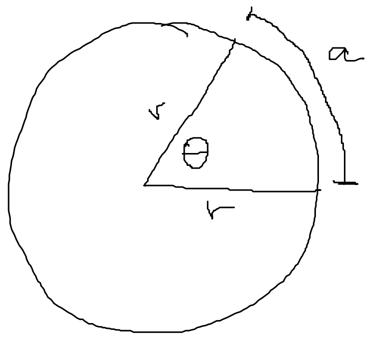
$$60^\circ \times \frac{2\pi}{360} \frac{\text{radians}}{\text{degrees}} = \frac{2\pi}{6} = \frac{\pi}{3} \text{ radians}$$

$$\frac{3\pi}{4} \text{ radians} \times \frac{360}{2\pi} \frac{\text{degrees}}{\text{radians}} = \frac{3 \cdot 360}{4 \cdot 2} = 135 \text{ degrees.}$$

For the tom cruise problem,

$$9.6 \text{ degrees} \times \frac{360}{2\pi} \frac{\text{degrees}}{\text{radians}} = \frac{19.2\pi}{360} \cong 0.1676 \text{ radians}$$

One thing that's nice about radians is that they relate directly to arc length.



circumference = $2\pi r$

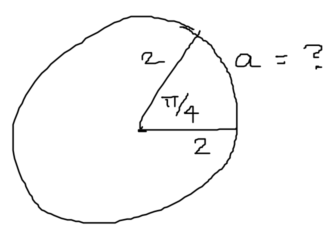
# of radians in that arc = 2π

$$\frac{\text{arc length } a}{\theta \text{ radians}} = \frac{2\pi r}{2\pi}$$

$$\Rightarrow \quad \frac{a}{\theta} = r \quad \Rightarrow a = r\theta$$

ex: given a circle of radius 3 and an arc subtendes the angle $\theta = \pi/4$ radians, what is the arclength?

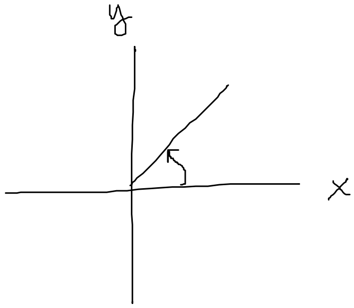
Draw a picture!



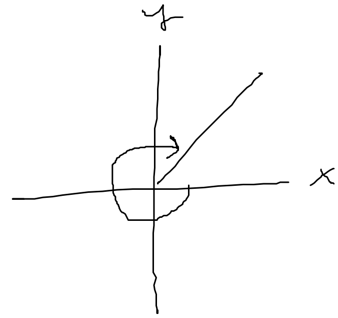
$$\frac{a}{\pi/4} = \frac{2\pi \cdot 2}{2\pi} \quad \Rightarrow \quad a = 2\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$$

When we draw angles in Cartesian coordinates (x-y coordinates) we represent the angle in standard position. Put the vertex of the angle at the origin, put its initial side on the positive x axis. A positive angle is found by rotating the initial side counterclockwise to the terminal side. A negative angle is found by rotating the initial side clockwise to the terminal side.

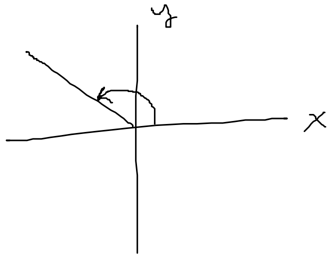
$\theta = \pi/4$



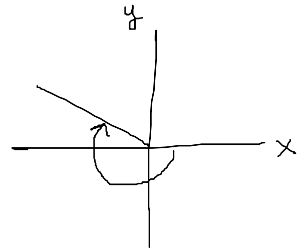
$\theta = -\frac{7\pi}{4}$



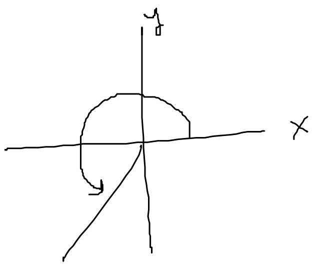
$\theta = \frac{3\pi}{4}$



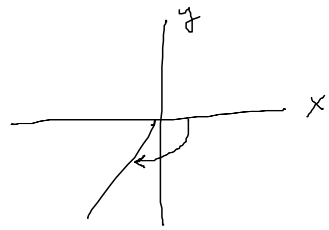
$\theta = -\frac{5\pi}{4}$



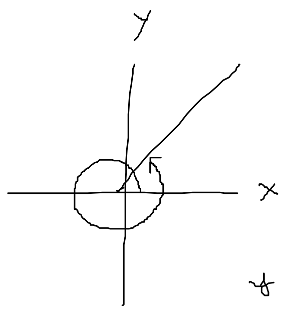
$\theta = \frac{5\pi}{4}$



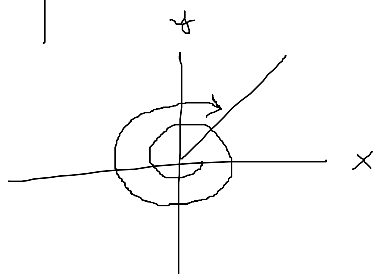
$\theta = \frac{-3\pi}{4}$



$\theta = \frac{9\pi}{4} = 2\pi + \frac{\pi}{4}$

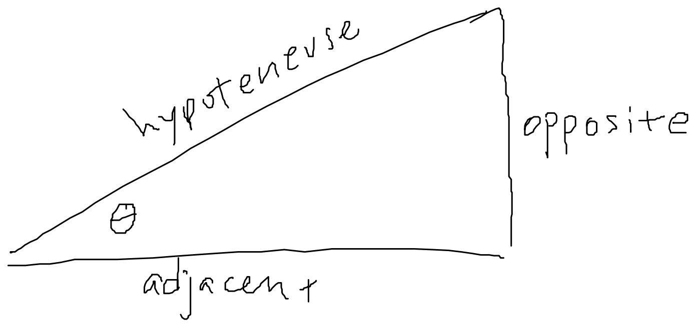


$\theta = -\frac{15\pi}{4} = -2\pi - \frac{7\pi}{4}$



Given any θ between 0 and 2π $\quad 0 \le \theta < 2\pi$
there are infinitely many θ in the real numbers that will yield the same terminal side when drawn in standard position

Now for trigonometry. First, we will consider acute angles: $0 < \theta < \pi/2$. The trig functions can be defined using a right triangle:



$$\sin(\theta) = \frac{opp}{hyp} \qquad \cos(\theta) = \frac{adj}{hyp} \qquad \tan(\theta) = \frac{opp}{adj}$$

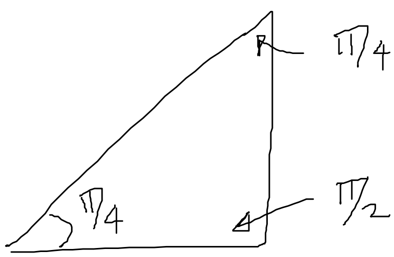
SOHCAHTOA

there are companion functions

$$\sin \sim \csc = 1/\sin = hyp/opp$$
$$\cos \sim \sec = 1/\cos = hyp/adj$$
$$\tan \sim \cot = 1/\tan = adj/opp$$

You must must must remember sin, cos, and tan and SOHCAHTOA. You then remember "csc, sec, cot are the inverses of sin, cos, tan, but which goes to which?" Then you remember "cos spelled backwards is soc which is like sec. So $\sec = 1/\cos$. Cot has a "t" in it, so does tan. So $\cot = 1/\tan$. Left over: $\csc = 1/\sin$."

consider $\theta = \pi/4$



you know opp = adj and you know

$$opp^2 + adj^2 = hyp^2 \qquad \text{(pythagorus' theorem)}$$

$$\Rightarrow \quad opp^2 + opp^2 = hyp^2$$

$$\Rightarrow \quad 2\,opp^2 = hyp^2$$

$$\Rightarrow \quad \left(\frac{opp}{hyp}\right)^2 = 1/2 \quad \Rightarrow \quad \frac{opp}{hyp} = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

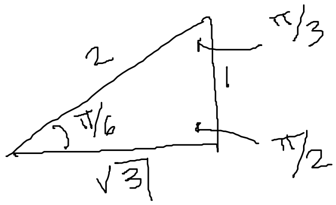
and $\cos\left(\frac{\pi}{4}\right) = \frac{adj}{hyp} = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}$ $\Rightarrow$

and $\tan\left(\frac{\pi}{4}\right) = \frac{opp}{adj} = 1$

$$\boxed{\begin{aligned} \sin\left(\frac{\pi}{4}\right) &= \frac{1}{\sqrt{2}} \\ \cos(\pi/4) &= \frac{1}{\sqrt{2}} \\ \tan(\pi/4) &= 1 \end{aligned}}$$

it follows

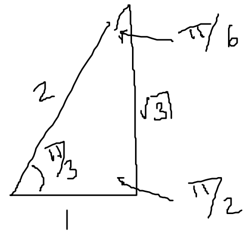
$$\boxed{\begin{aligned} \csc(\pi/4) &= \sqrt{2} \\ \sec(\pi/4) &= \sqrt{2} \\ \cot(\pi/4) &= 1 \end{aligned}}$$



$\sin(\pi/6) = 1/2$ $\qquad$ $\csc(\pi/6) = 2$

$\cos(\pi/6) = \sqrt{3}/2$ $\quad \Rightarrow \quad$ $\sec(\pi/6) = 2/\sqrt{3}$

$\tan(\pi/6) = 1/\sqrt{3}$ $\qquad$ $\cot(\pi/6) = \sqrt{3}$



$\sin(\pi/3) = \sqrt{3}/2$ $\qquad$ $\csc(\pi/3) = 2/\sqrt{3}$

$\cos(\pi/3) = 1/2$ $\quad \Rightarrow \quad$ $\sec(\pi/3) = 2$

$\tan(\pi/3) = \sqrt{3}$ $\qquad$ $\cot(\pi/3) = 1/\sqrt{3}$

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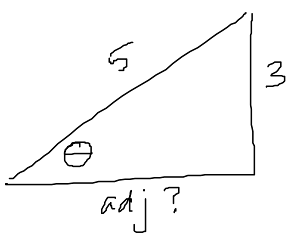
ex #29 find the five remaining trigonometric ratios.

$\sin(\theta) = 3/5 \qquad 0 < \theta < \pi/2$



$\text{adj}^2 + \text{opp}^2 = \text{hyp}^2$

$\Rightarrow \text{adj}^2 + 9 = 25$

$\Rightarrow \text{adj}^2 = 16 \Rightarrow \text{adj} = 4$

$\cos(\theta) = 4/5$

$\tan(\theta) = 3/4$

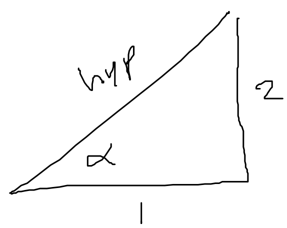
$\csc(\theta) = 5/3$

$\sec(\theta) = 5/4$

$\cot(\theta) = 4/3$

#30 find the remaining trig ratios.

$\tan(\alpha) = 2 \qquad 0 < \alpha < \pi/2$

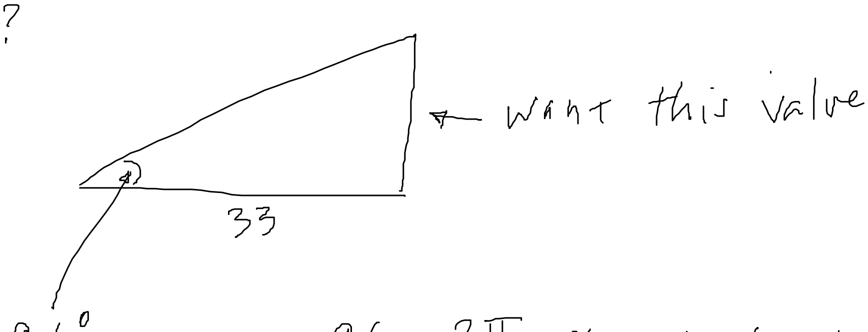


$adj^2 + opp^2 = hyp^2$

$4 + 1 = hyp^2 \Rightarrow hyp = \sqrt{5}$

$$\begin{cases} \sin(\alpha) = 2/\sqrt{5} \\ \cos(\alpha) = 1/\sqrt{5} \\ \tan(\alpha) = 2 \end{cases} \Rightarrow \begin{cases} \csc(\alpha) = \sqrt{5}/2 \\ \sec(\alpha) = \sqrt{5} \\ \cot(\alpha) = 1/2 \end{cases}$$

Tom Cruise ?



$9.6 \times \frac{2\pi}{360} \cong .1676$ radians

$$\tan\left(\frac{19.2\pi}{360}\right) = \frac{\text{T.C.'s height}}{33}$$

$$\Rightarrow \text{T.C.'s height} = 33\tan\left(\frac{19.2\pi}{360}\right) \cong 5.5815$$

$$= 5'7''$$

What if we'd had the distance off by 1 foot?

$$32 \tan\left(\frac{19.2 \times \pi}{360}\right) \cong 5'5''$$

$$34 \tan\left(\frac{19.2\pi}{360}\right) \cong 5'9''$$

What if we'd had the angle off by ½ a degree?

$9.6^\circ + .5^\circ = 10.1$

$$33 \tan\left(\frac{20.2\pi}{360}\right) \cong 5'11''$$

$9.6 - .5 = 9.1$

$$33 \tan\left(\frac{18.2\pi}{360}\right) \cong 5'3''$$

draw some triangles and convince yourself these are reasonable results!! i.e. if you got the distance too long he'd appear taller than he really is, etc.