If A is an $n \times n$ diagonalizable matrix then the general solution of $\vec{X}' = A\,\vec{X}$ is $\vec{X}(t) = \sum_{k=1}^{n} c_k\, e^{\lambda_k\, t} \vec{v}_k$ where $(\lambda_k, \vec{v}_k)$ are n eigenvalue-eigenvector pairs of A, chosen so that $\{\vec{v}_1, \vec{v}_k, \ldots \vec{v}_n\}$ is a linearly independent set.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} = (ad - bc)\, I$$

The vectors E_1 and E_2 are

$$E_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad E_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

Equation-solving techniques

- To solve an equation of the form $\frac{dx}{dt} = f(t)\varphi(x)$, rewrite as $\frac{dx}{\varphi(x)} = f(t)dt$. Then take the integral. Don't forget the integration constant. In addition, do not forget to check the case when $\varphi(x) = 0$.
- To solve a first-order linear equation of the form $x' + p(t)x = q(t)$, first multiply the equation by $\mu(t)$. Then choose a $\mu(t)$ so that $\mu'(t) = \mu(t)p(t)$. With this choice of $\mu(t)$, the ODE becomes $(\mu(t)x(t))' = \mu(t)q(t)$. Integrate the equation with respect to t and, if possible, solve for $x(t)$.
- $X' = AX + G(t)$ with $X(t_0) = X_0$ has solution

$$X(t) = e^{tA} X_0 + \int_{t_0}^{t} e^{(t-s)A} G(s)\, ds$$

- $X' = A(t)X + G(t)$ with $X(t_0) = X_0$ has solution

$$X(t) = \Psi(t)\Psi(t_0)^{-1} X_0 + \int_{t_0}^{t} \Psi(t)\Psi(s)^{-1} G(s)\, ds$$

  where the columns of $\Psi(t)$ are n linearly independent solutions of $X' = A(t)X$.
- To solve $y'' + q(t)y' + r(t)y = g(t)$, first find linearly independent solutions y_1 and y_2 of $y'' + q(t)y' + r(t)y = 0$. A particular solution is $y_p(t) = u_1(t)y_1(t) + u_2(t)y_2(t)$ where

$$u_1(t) = -\int \frac{y_2(t)g(t)}{W(t)}\, dt, \qquad u_2(t) = \int \frac{y_1(t)g(t)}{W(t)}\, dt$$

  Where $W(t) = \det([y_1(t), y_2(t); y_1'(t), y_2'(t)])$.