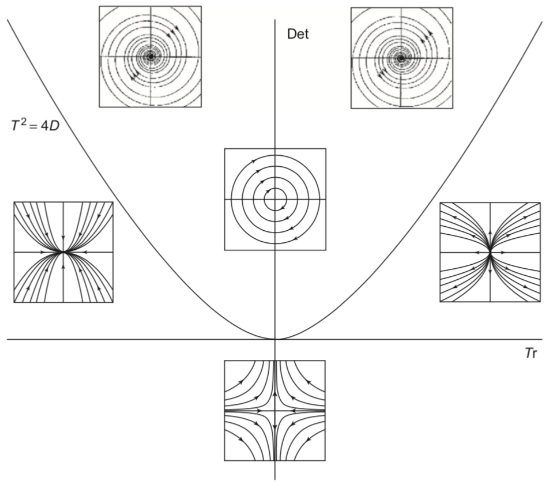

Figure 0.1: The trace-determinant plane. Warning: each point in the plane represents infinitely many systems $X' = AX$. Angles may change, directions of rotation may change, and so forth!

If A is an $n \times n$ diagonalizable matrix then the general solution of $\vec{X}' = A\,\vec{X}$ is $\vec{X}(t) = \sum_{k=1}^{n} c_k\, e^{\lambda_k\, t}\vec{v}_k$ where $(\lambda_k, \vec{v}_k)$ are n eigenvalue-eigenvector pairs of A, chosen so that $\{\vec{v}_1, \vec{v}_k, \ldots \vec{v}_n\}$ is a linearly independent set.

Equation solving techniques

- To solve an equation of the form $\frac{dx}{dt} = f(t)\varphi(x)$, rewrite as $\frac{dx}{\varphi(x)} = f(t)dt$. Then take the integral. Don't forget the integration constant. In addition, do not forget to check the case when $\varphi(x) = 0$.
- To solve a first-order linear equation of the form $x' + p(t)x = q(t)$, first multiply the equation by $\mu(t)$. Then choose a $\mu(t)$ so that $\mu'(t) = \mu(t)p(t)$. With this choice of $\mu(t)$, the ODE becomes $(\mu(t)x(t))' = \mu(t)q(t)$. Integrate the equation with respect to t and, if possible, solve for $x(t)$.

Zoology of planar linear systems $X' = AX$ when $\det(A) \neq 0$.

name	eigenvalues
saddle	$\lambda_1 < 0 < \lambda_2$
sink	$\lambda_1 < \lambda_2 < 0$
source	$0 < \lambda_1 < \lambda_2$
center	$\lambda_j = \pm i\beta,\ \beta \neq 0$
spiral sink	$\lambda_j = \alpha \pm i\beta,\ \alpha < 0,\ \beta \neq 0$
spiral source	$\lambda_j = \alpha \pm i\beta,\ \alpha > 0,\ \beta \neq 0$
stable proper node	$\lambda_1 = \lambda_2 < 0$ and A is diagonalizable
unstable proper node	$0 < \lambda_1 = \lambda_2$ and A is diagonalizable
stable improper node	$\lambda_1 = \lambda_2 < 0$ and A is not diagonalizable
unstable improper node	$0 < \lambda_1 = \lambda_2$ and A is not diagonalizable