1. **(1 point)** `Library/Rochester/setDiffEQ13Systems1stOrder/ur_de_13_16.pg`

Multiplying the differential equation

$$\frac{df}{dt} + af(t) = g(t),$$

where a is a constant and $g(t)$ is a smooth function, by e^{at}, gives

$$e^{at}\frac{df}{dt} + e^{at}af(t) = e^{at}g(t),$$

$$\frac{d}{dt}\left(e^{at}f(t)\right) = e^{at}g(t),$$

$$e^{at}f(t) = \int e^{at}g(t)dt,$$

$$f(t) = e^{-at}\int e^{at}g(t)dt.$$

Use this to solve the initial value problem

$$\frac{dx}{dt} = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} x,$$

with $x(0) = \begin{bmatrix} 4 \\ -4 \end{bmatrix}$,

i.e. find first $x_2(t)$ and then $x_1(t)$.

$x_1(t) =$ ________________,
$x_2(t) =$ ________________.

Correct Answers:

- `1*-4/(-2 - -1) * e^(-2*t) + (4 - 1*-4/(-2 - -1)) * e^(-1*t)`
- `-4 * e^(-2*t)`

2. **(1 point)** `Library/Rochester/setDiffEQ13Systems1stOrder/ur_de_13_6.pg`

Match the differential equations and their vector valued function solutions:

It will be good practice to multiply at least one solution out fully, to make sure that you know how to do it, but you can get the other answers quickly by process of elimination and just multiply out one row element.

___1. $y'(t) = \begin{bmatrix} -86 & 218 & -160 \\ 73 & -49 & 80 \\ 111 & -138 & 165 \end{bmatrix} y(t)$

___2. $y'(t) = \begin{bmatrix} -32 & 49 & -23 \\ -64 & 78 & -34 \\ 64 & -78 & 34 \end{bmatrix} y(t)$

___3. $y'(t) = \begin{bmatrix} -13 & -2 & 3 \\ -15 & -18 & 5 \\ -33 & -18 & 7 \end{bmatrix} y(t)$

A.
$$y(t) = \begin{bmatrix} -1 \\ 0 \\ -3 \end{bmatrix} e^{-4t}$$

B.
$$y(t) = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix} e^{45t}$$

C.
$$y(t) = \begin{bmatrix} -1 \\ -3 \\ -5 \end{bmatrix}$$

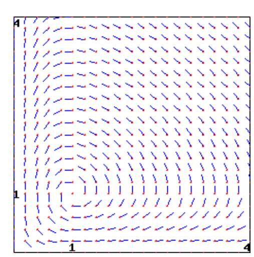
Correct Answers:

- `B`
- `C`
- `A`

3. **(1 point)** `Library/Michigan/Chap11Sec8/Q05.pg`

Let w be the number of worms (in millions) and r the number of robins (in thousands) living on an island. Suppose w and r satisfy the following differential equations, which correspond to the slope field shown below.

$$\frac{dw}{dt} = w - wr, \qquad \frac{dr}{dt} = -r + wr.$$



Assume $w = 4$ and $r = 4$ when $t = 0$.

Does the number of worms increase, decrease, or stay the same at first?

- ?
- increases
- decreases
- stays the same

Does the number of robins increase, decrease, or stay the same at first?

- ?
- increases
- decreases
- stays the same

What happens in the long run?

- ?
- w and r both go to zero
- w goes to zero and r increases to infinity
- w increases to infinity and r goes to zero
- w and r both go to stable long-term values
- w and r oscillate

Correct Answers:

- `decreases`
- `increases`
- `w and r oscillate`

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4. **(1 point)** `Library/METU-NCC/Diff_Eq/2x2-system_soln-cx.pg`

Suppose that the matrix A has the following eigenvalues and eigenvectors:

$\lambda_1 = 5i$ with $\vec{v}_1 = \begin{bmatrix} 1 \\ -5+i \end{bmatrix}$.

and

$\lambda_2 = -5i$ with $\vec{v}_2 = \begin{bmatrix} 1 \\ -5-i \end{bmatrix}$.

Write the solution to the linear system $\vec{r}\,' = A\vec{r}$ in the following forms.

A. In eigenvalue/eigenvector form:

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = c_1\left(\begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}\_\_ - \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}\_\_\right)e^{\_\_t} + c_2\left(\begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}\_\_ + \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}\_\_\right)e^{\_\_t}$$

B. In fundamental matrix form:

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \_\_\_\_ & \_\_\_\_ \\ \_\_\_\_ & \_\_\_\_ \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

C. As two equations: *(write "c1" and "c2" for c_1 and c_2)*

$x(t) =$ ____________________

$y(t) =$ ____________________

Note: if you are feeling adventurous you could use other eigenvectors like $4\vec{v}_1$ or $-3\vec{v}_2$.

Correct Answers:

- ```
  <table border='0'><tr>
      <td> \(c_1\ \Biggl( \biggl\lbrack\) </td>
      <td><table border='0' cellspacing='10'><tr><td> 1 </td
      <td> \(\biggr\rbrack\ cos(5*t) - \biggl\lbrack \)</td>
      <td><table border='0' cellspacing='10'><tr><td> 0 </td
      <td> \(\biggr\rbrack\ sin(5*t) \Biggr) e^{0 t} \ \ +
      <td><table border='0' cellspacing='10'><tr><td> 1 </td
      <td> \(\biggr\rbrack\ sin(5*t) - \biggl\lbrack \)</td>
      <td><table border='0' cellspacing='10'><tr><td> 0 </td
      <td> \(\biggr\rbrack\ cos(5*t) \Biggr) e^{0 t} \) </td
    </tr></table>
  ```
- ```
  <table border='0'><tr>
      <td> \( \biggl\lbrack \)</td>
      <td><table border='0' cellspacing='10'>
            <tr><td> cos(5*t) \  </td><td> \  sin(5*t)</td>
            <tr><td> -[5*cos(5*t)+sin(5*t)] \  </td><td> \
      <td> \(\biggr\rbrack\ \biggl\lbrack\)</td>
      <td><table border='0' cellspacing='10'><tr><td> \(c_1\
                                                 <tr><td> \(c_2\
      <td> \(\biggr\rbrack\) </td>
     </tr></table>
  ```
- ```
  <table border='0'>
          <tr><td style='text-align:left'>\(x(t) = \) c1*co
          <tr><td style='text-align:left'>\(y(t) = \) [cos(
  ```

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5. **(1 point)** `Library/274/systems/prob92.pg`

Find the equilibrium solution for

$$x_1'(t) = -6.2 + 1.1x_1 - 0.8x_2$$

$$x_2'(t) = -13.8 + 2.1x_1 - 1.2x_2$$

$$x_1(0) = 11;\ x_2(0) = 4$$

Equilibrium: $x_1^e =$ __________,
$x_2^e =$ __________.

[Note– you may want to view a **phase plane plot** (right click to open in a new window).]

[?] 1. Describe the trajectory.

[?] 1. What kind of interaction do we observe?

Correct Answers:

- `10`
- `6`
- `Spiral inward counterclockwise`
- `Predator-prey`
  ```