VECTOR CALC STUFF - INTEGRALS, GRAD, DIV, CURL

Problem 1. (Jan 11 #1) Compute $\int_{S^2} x_1^2 x_2^2 dS(x)$. Here S^2 is the unit sphere in $\mathbb{R}^3$, $x = (x_1, x_2, x_3)$ and dS is the surface area element.

Problem 2. (Jan 09 #2) Compute $\int_L (y-z)dx + (z-x)dy + (x-y)dz$ where L is the curve given by the intersection of the two surfaces:

$$\begin{aligned} x^2 + y^2 + z^2 &= a^2 \\ x + y + z &= 0 \end{aligned}$$

with counterclockwise orientation viewed from the positive x-axis.

Problem 3. (Sept 07 #5 Part I) Evaluate the integral $\int_S (x^2 + y^2) d\sigma$, where S is the sphere of radius 1 centered at $(0,0,0)$ and $d\sigma$ is surface area.

Problem 4. (Sept 10 #5) Part I: For every positive integer n, find $m(n)$ such that the following integral is finite for $m > m(n)$:

$$\int_{\mathbb{R}^n} \frac{dx_1 \dots dx_n}{1 + \sum_{i=1}^n |x_i|^m}$$

Part II: Evaluate the integral $\int_{\mathbb{R}^3} \frac{dx_1 dx_2 dx_3}{\left(1 + \sum_{i=1}^3 x_i^2\right)^2}$.

Problem 5. In $\mathbb{R}^3$ let C be the circle in the xy plane with radius 2 and the origin as center, i.e. $C = \{x^2 + y^2 = 4, z = 0\}$. Let Ω consist of all the points $(x, y, z) \in \mathbb{R}^3$ whose distance to C is at most 1. Compute:

$$\int_\Omega |x| dx dy dz$$

Problem 6. Let $f : [a, b] \to \mathbb{R}$ be a continuously differentiable function and S be the surface in $\mathbb{R}^3$ obtained by revolving the curve $y = f(x)$ around the x axis. Determine the surface area of S.