MAT1347 Topics in Geometry, 2020-21

Shifted Symplectic geometry

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Time: TBA
Room: TBA

Shifted symplectic geometry goes back to methods developed by physicists for understanding field theories with various types of symmetries, redundancies, and degeneracies. These methods were developed in the physics literature by Fadeev-Popov, Becchi-Rouet-Stora-Tyutin, Batalin-Vilkoviskiy, and many others. Recently it has been recognized and developed formally, to great effects, in the mathematics literature, by Pantev-Toën-Vaquié-Vezzosi and others. The subject involves symplectic structures on higher and derived stacks or generalized spaces. The grading associated to the generalized space allows a symplectic structure to be "shifted" by an integer – the zero shift corresponds to a usual symplectic structure. For shift equal to one, these structures are closely related to Poisson geometry, and for shift equal to two, to gerbes, Courant algebroids, and Dirac structures. Shift minus one is particularly relevant to physics. In this course we will develop the subject in a concrete way, focusing on low shift numbers and the relation to other parts of mathematics and physics, including as many of the following topics as possible.

- Categories and simplicial sets
- Simplicial manifolds and their equivalence classes: Stacks
- Lie groupoids
- The cosimiplicial de Rham complex, proof of the descent to stacks
- Double Lie groupoids
- The tangent complex as a Double vector bundle and as a chain complex
- The definition of a shifted symplectic form
- The example of the classifying stack of G-bundles for a quadratic Lie group
- Quasi-Hamiltonian spaces, reduction, and the AKSZ TQFT.
- The Atiyah-Bott symplectic form on the moduli space of flat connections on a surface.