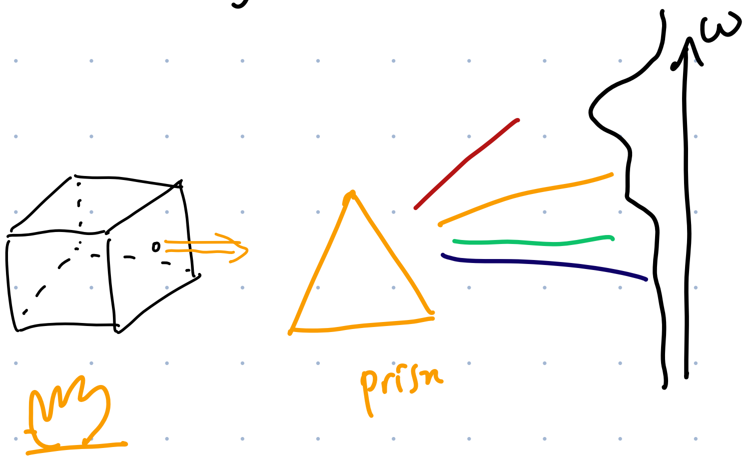
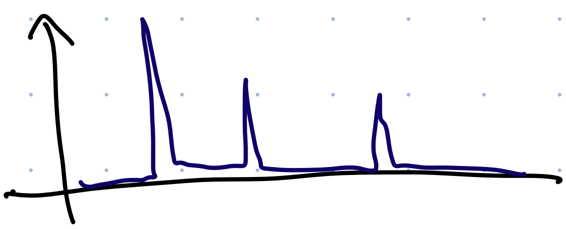
Blackbody Radiation



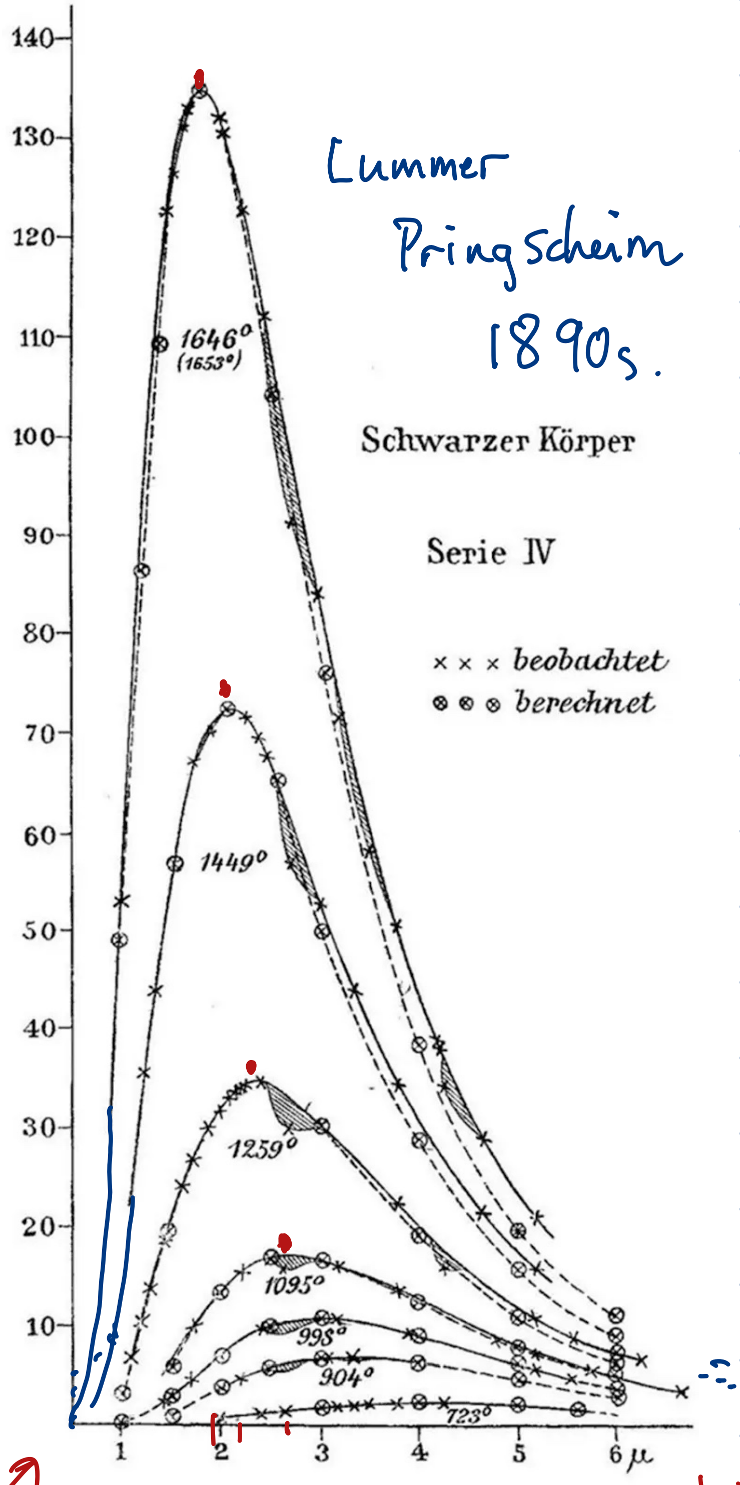
Pure Gas (Ar, Kr, Ne, ...)



Remove everything: empty of matter

⇒ Blackbody spectrum

Lummer Pringsheim 1890s.



wavelength

Kirchhoff: Q.: Derive this

↑ advisor

Planck: Answer: $\boxed{I \propto \omega^2 \frac{h\omega}{e^{h\omega}-1}}$ Planck Law.

Math model: Collection of Harmonic oscillators with different frequencies ω.

$$H(\omega) = \frac{\omega}{2}(x^2+p^2)$$

Main task: compute density of states $\Omega(E)\,dE$

\# of oscill. of freq $\omega \;\propto\; \omega^2$ (2-sphere in momentum space).

using Fourier decomposition: waves in 3d space

$$e^{i\vec{k}\cdot\vec{x}} \qquad \vec{k} \in \mathbb{R}^3$$

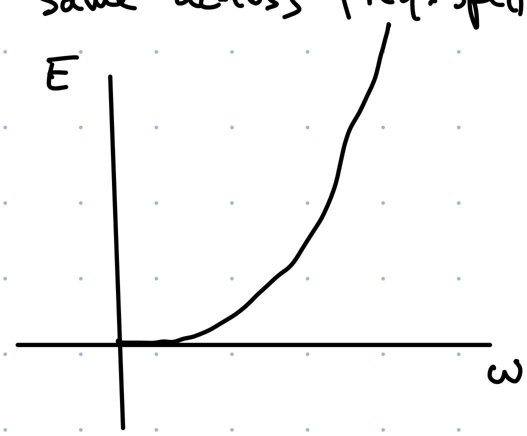
frequency $|\vec{k}| = \omega$

for Harm. Osc.: $\Omega(E) \propto E^N = E^{\omega^2}$

$$\frac{1}{T} = \frac{\partial \log \Omega}{\partial E} = \omega^2 \cdot \frac{1}{E}$$

require that temperature same across freq. spect. (assumption of equil.)

$$\Rightarrow E \propto \omega^2 T$$



Ultraviolet catastrophe: higher freq. are always preferred

Idea: assume that energy is only emitted in quanta, and that this quantum

$$\Delta = h\omega$$

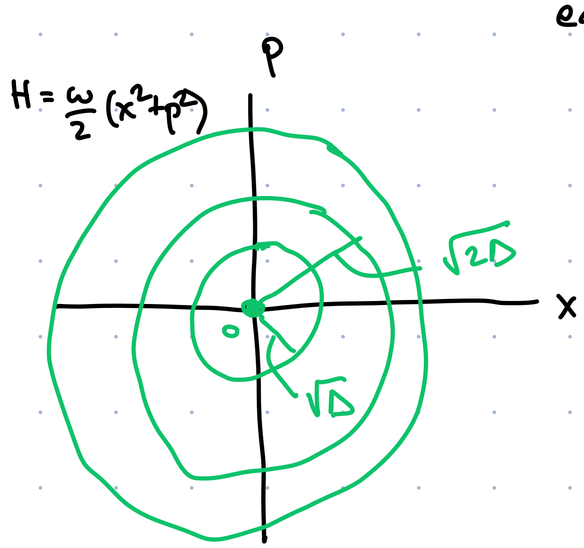
is proportional to frequency. (Einstein photon freq ω has energy $h\omega$)

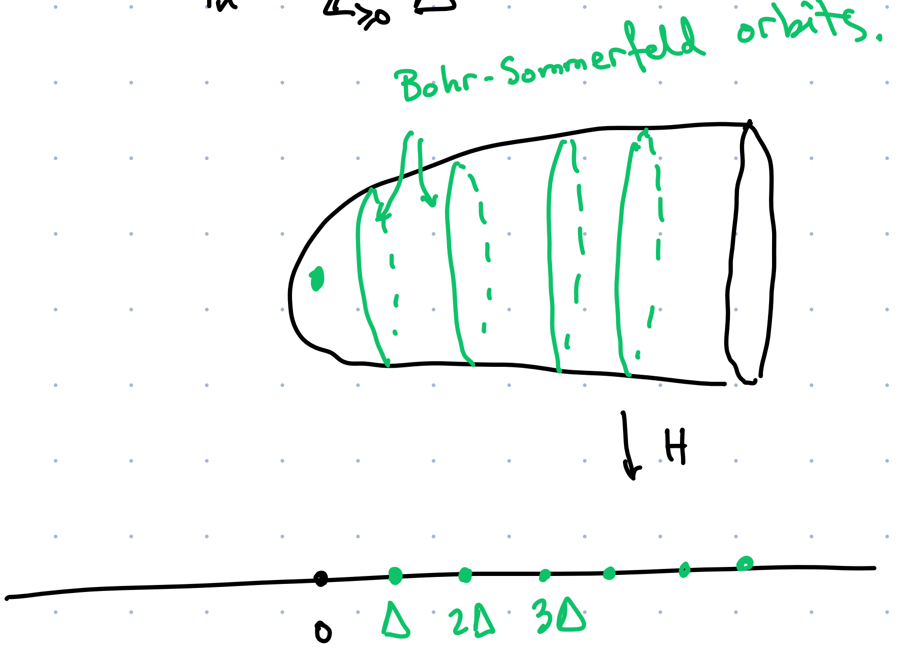
(Makes high freq. states more difficult to populate).

Consequences: fix frequency ω

$N(\omega)$ oscillators of freq. ω

each osc. may only have energy levels in $\mathbb{Z}_{\geq 0} \cdot \Delta$





Total # of quanta at freq ω is P

The P quanta must be distributed among N oscillators

$$\boxed{\begin{array}{l}\#\text{ of states} \\ \Omega_\omega(E = P\Delta) \;=\; ?\end{array}}$$

$$\overbrace{\bullet\;\bullet\;|\;\bullet\;\bullet\;\bullet\;\bullet\;|\;\bullet\;\bullet\;|\;\bullet}^{P\text{ quanta}}$$

1 2 3

# of symbols

$P+N-1$

choose $N-1$ to be partition walls.

# of such states is $\binom{P+N-1}{N-1}$

Note: if we view quanta as particles we treat them as indistinguishable.

but the oscillators are ordered

$$\simeq \frac{(P+N)!}{N!\,P!}$$

Stirling formula.

$$x! \sim \frac{x^x}{e^x}$$

$$\log x! = x\log x - x$$

$$\log \Omega_\omega(E) = \log(P+N)! - \log N! - \log P!$$

$$\overset{\text{Stirling}}{=} (P+N)\log(P+N) - \cancel{(P+N)}$$
$$- (N\log N - \cancel{N})$$
$$- (P\log P - \cancel{P})$$

$$= (P+N)\log(P+N) - N\log N - P\log P.$$

$$= N\left[\left(\frac{P}{N}+1\right)\left(\log\left(\frac{P}{N}+1\right) + \log N\right)\right.$$
$$- \log N$$

cancel

$$\left[\quad - \frac{P}{N}\left(\log\frac{P}{N} + \log N\right) \right]$$

$$P = \frac{E}{\Delta}$$

$$\frac{P}{N} = \frac{\bar{E}}{\Delta} \qquad \bar{E} = E/N$$

mean energy

$$S_\omega = N\left(\left(\frac{\bar{E}}{\Delta}+1\right)\log\left(\frac{\bar{E}}{\Delta}+1\right) - \frac{\bar{E}}{\Delta}\log\frac{\bar{E}}{\Delta} \right)$$

$$\frac{1}{T} = \frac{\partial S_\omega}{\partial E} = \frac{1}{\Delta}\left(\log\left(\frac{\bar{E}}{\Delta}+1\right) - \log\frac{\bar{E}}{\Delta} \right)$$

$$\boxed{\frac{1}{T} = \frac{1}{\Delta}\log\left(\frac{\frac{\bar{E}}{\Delta}+1}{E/\Delta}\right)}$$

can now assume that sectors w/ freq ω have same temperature

$$e^{\frac{\Delta}{T}} = \frac{\frac{\bar{E}}{\Delta}+1}{E/\Delta}$$

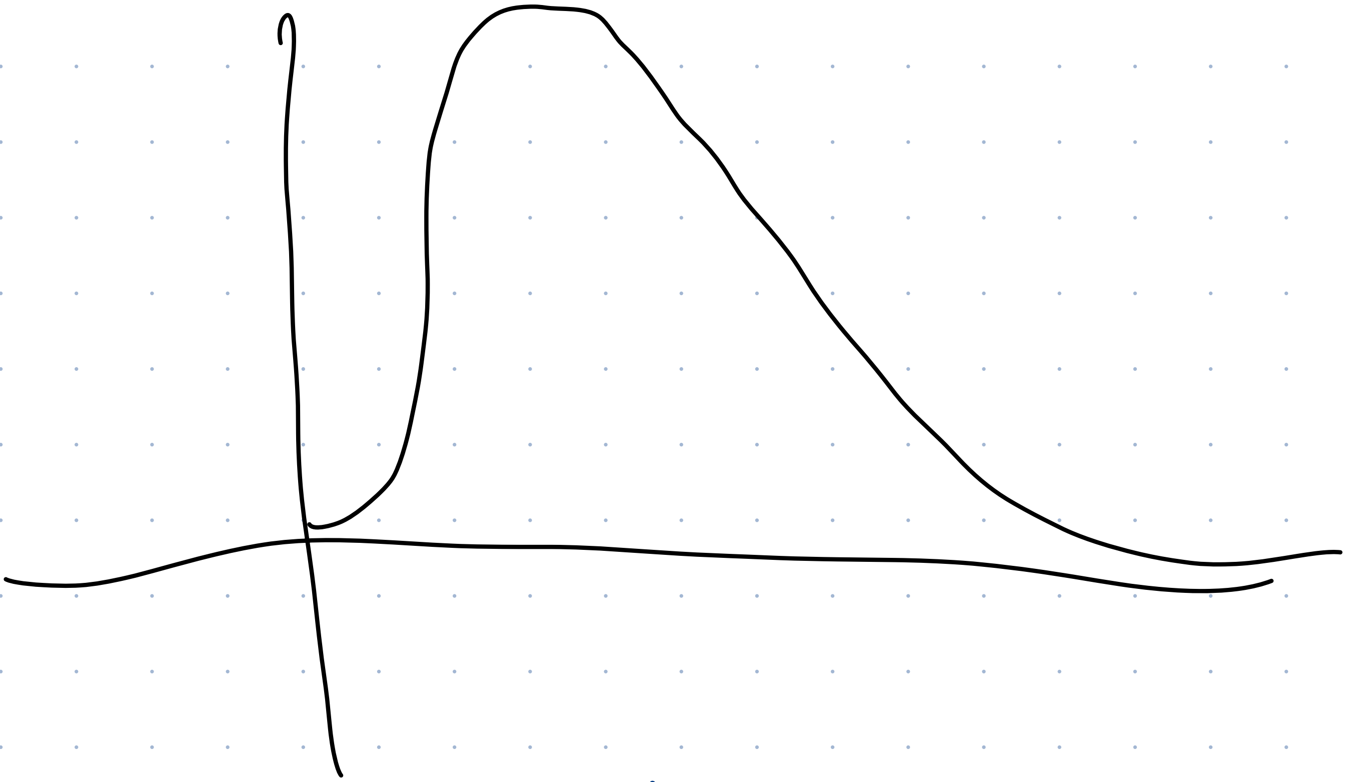
$$\bar{E} = \frac{\Delta}{e^{\frac{\Delta}{T}} - 1}$$

$$\boxed{E = N \cdot \frac{\Delta}{e^{\frac{\Delta}{T}} - 1}}$$

$$E(\omega) = N(\omega)\,\frac{\hbar\omega}{e^{\frac{\hbar\omega}{T}}-1}$$

$$\boxed{E(\omega) \propto \underset{(\dim 3)}{\omega^2} \cdot \frac{\hbar\omega}{e^{\frac{\hbar\omega}{T}}-1}}$$

Planck Law.



<u>Low freq:</u> $e^{\hbar\omega/T} = 1 + \frac{\hbar\omega}{T} + \dots$

$$E(\omega) \propto \omega^2 \, \frac{\cancel{\hbar}\omega}{\cancel{1+}\frac{\hbar\omega}{T}\cancel{-1} + \text{h.o.t.}}$$

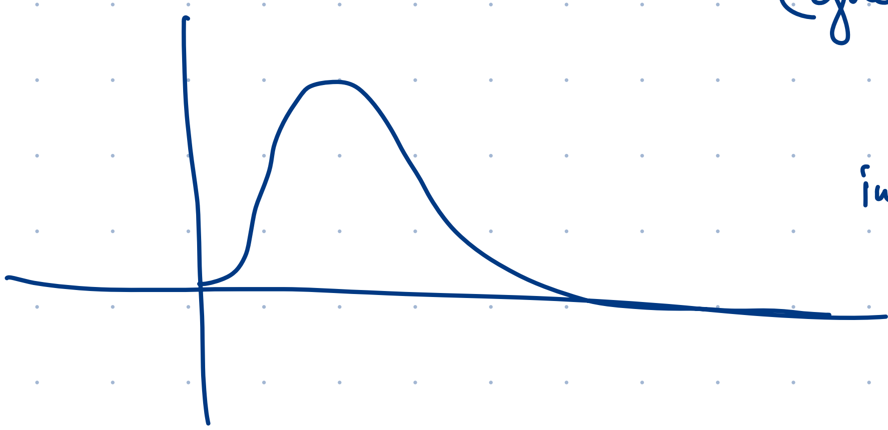
$$= \omega^2 T$$

Rayleigh-Jeans
valid at

valid at
Low ω.

high freq: $E(\omega) \propto \omega^3 e^{-\hbar\omega/T}$

Wien Law
(guessed)



invalid at low freq.

R-J $\longleftarrow$ Planck $\longrightarrow$ Wien Law

low ω

$$\omega^2 T \longleftarrow \omega^3 \frac{1}{e^{\frac{\hbar\omega}{T}} - 1} \longrightarrow \omega^3 e^{-\hbar\omega/T}$$

---

quanta of radiation w energy $\hbar\omega$

are <u>photons</u>

we are assuming that photons are
- indistinguishable
- permutation invariant

$\Rightarrow$ **Bosons** $\Rightarrow$

Planck distrib. is a special case of

Bose-Einstein statistics.

(as opposed to Fermi-Dirac).

(two fermions cannot be in same state)