MGSA

The Newsletter of the Mathematics Graduate Student Association

Issue 2 April 2025

It's been a while!

Goodness, is it April already? Time flies when this semester is packed with events like the Faculty-Student Mixer, the Bird's Eye Conference, the Math+CS Symposium, and even our first-ever election for Secretary. (Vote for who you think should take my job!) Not to mention, of course, all of our usual research and teaching. Amid all that hustle and bustle, I hope this issue can bring you a bit of lightheartedness and maybe even a little food for thought.

Speaking of research, we often think of research as proving new theorems. But what about the process of searching for a proof, or of communicating a proven result? This issue, Waleed Qaisar discusses with Fields Medallist Alain Connes about his creative process in an interview (excerpt on page 10). In another article, Kevin Santos explores mathematical writing through a literary lens, and asks how an awareness of mathematical rhetoric could impact how we write about math (page 5).

Need a break from all that math? Check out the comic, crossword, and puzzles (pages 2, 8, and 6). We'd love to see your solutions to the puzzles and crossword!

Last issue, we posted some questions from anonymous students about topics such as when to give up on research problems. You can read anonymous responses from your peers in Asking for a Friend (page 12), and answer some new questions that were submitted. You can ask a question too!

We hear your struggles at the MGSA, and we're working behind the scenes to support you. Learn how we found faculty to offer the Topics Courses that you requested in The Inside Scoop (page 3).

Enjoy reading, and tell us your thoughts in the feedback survey. Godspeed for your finals!

Herng Yi Cheng
MGSA Secretary □

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View the digital version to take part in the anonymous Q&A, give feedback, and more.

www.math.utoronto.ca/mgsa/newsletter

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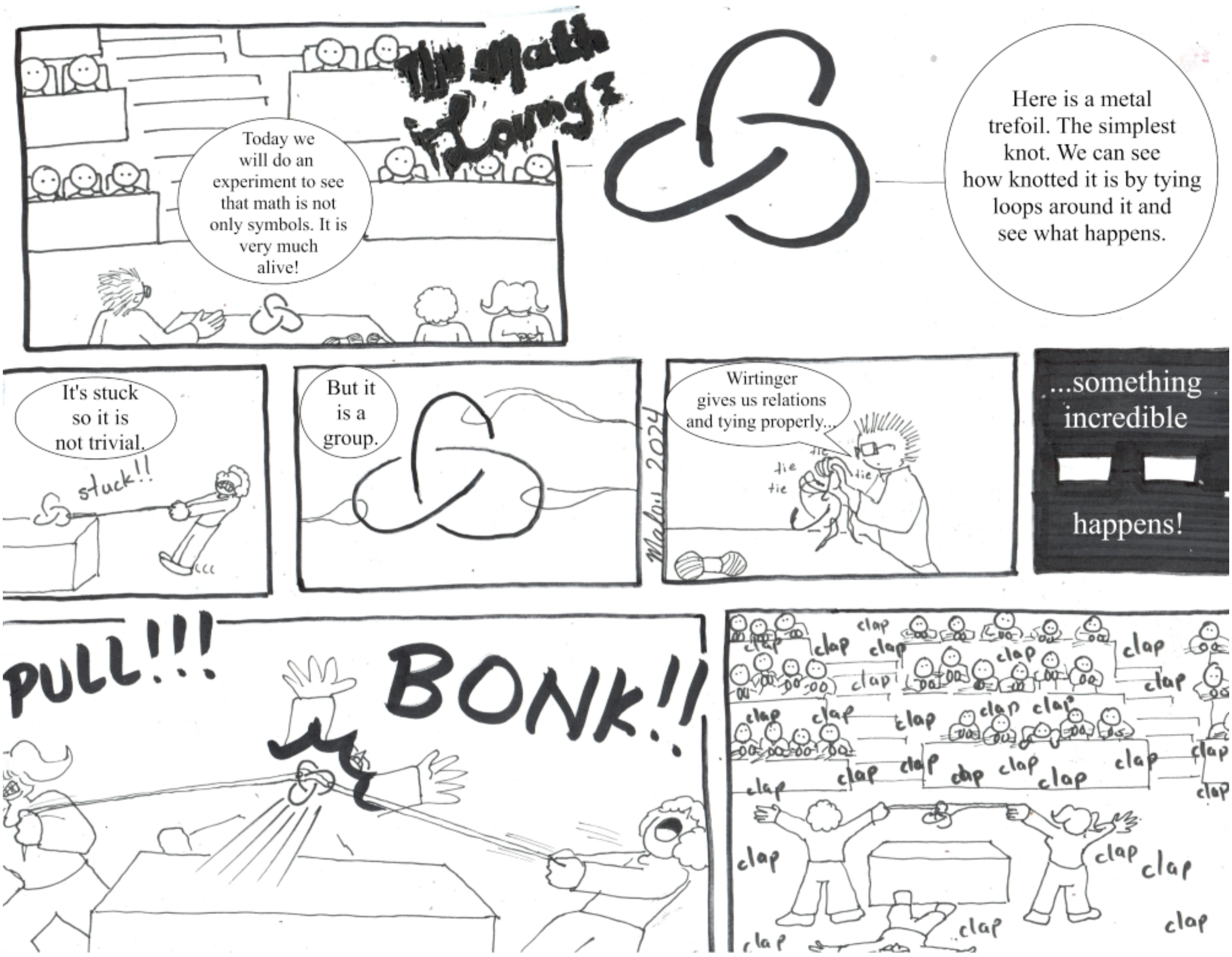
by Malors Espinosa Lara



The Inside Scoop

Updates on the MGSA's Advocacy

by Narmada Varadarajan (President), James Munday (Vice President), and
Alisa Chistopolskaia (Academic Chair)

The Faculty-Student Mixer

Group photo for the Bird's Eye Conference
(Photo by Sara Sivan)

Events

There were two big events in the last month in the math department. The first was the Faculty Student Mixer, on February 28th, which brought together faculty and students for an afternoon of socialization. There were over 25 faculty members and over 60 students in attendance. Thanks again to our faculty sponsors Dror Bar-Natan, Jeremy Quastel, Arul Shankar, and Ila Varma for making this event possible! And thanks to all the faculty and students who came and made this a successful event!

The other big event recently was the Birds Eye Conference. This is not an MGSA event, and we owe many thanks to this year's organizers, Cameron Martin, Fardin Syed, and William Verrault! The conference had 81 registered grad student attendees, and even some prospective students, and featured over 30 talks from all corners of the department. Thanks to everyone who came out!

Advocacy

The Course Planning Committee has been working with the department on implementing courses requested by graduate students. This Fall, we collected feedback from graduate students about core courses, qualifying exams, and topics courses. In the last issue we shared with you an update about qualifying exams. Since then, there was a Departmental Graduate Committee meeting that we attended. The Department can offer 20 courses, outside of core courses and cross-listed courses. Out of these 20 courses, 12 were the courses that students requested in the Academic Survey! They include intermediate courses on Algebraic geometry taught by Daniel Litt and Michael Groechenig and MAT1800 taught by Adam Stinchcombe. We want to underline that the major part of the work had to be done prior to the meeting itself: to get these 12 proposed courses we had to ask 29 Faculty members to propose a course! Currently we are surveying Faculty on the core courses they would advise to take before working with them. We are also working on something else for the next year - stay tuned for the news!

□

The Rhetoric of Mathematical Writing

by Kevin Santos

Many mathematicians are drawn to the subject because of its aesthetic appeal; in the words of G. H. Hardy, "The mathematician's patterns, like the painter's or the poet's, must be beautiful; the ideas like the colours or the words, must fit together in a harmonious way." But while some may believe that mathematics exists as a platonic ideal of beauty and perfect logic, what does that mean for those who must write about it and communicate these patterns, in the context of teaching, exposition, or paper-writing?

Mathematical writing, like any genre of writing, comes with its own conventions, and it's important to understand these conventions as well as when to break them.

Writers in any field must make decisions about how to organize and communicate their ideas in order to best convey them. Mathematical writing presents its own unique challenges, such as the task of presenting complex or abstract ideas in an understandable way. In response to this, mathematicians have developed their own idiosyncratic style and approach to writing. Understanding the rhetoric of mathematical writing helps us become better writers by drawing our attention to the decisions that we must make when writing, choices that we might not even be conscious of. We can also improve as readers by recognizing and unpacking the underlying assumptions behind the papers we read.

In his article "Stylizing Rigor; or, Why Mathematicians Write So Well", Alex Csizsar describes some of the philosophies that guide mathematical writing. He notes that mathematicians seek to distinguish their writing from that of natural scientists; for example, papers in the natural sciences detail the process of the scientific method: hypothesis, methods, and results. However, mathematical papers and proofs do not generally reflect the actual process of generating mathematical research. The hypotheses or questions that might have initially motivated the research tend to be omitted, along with the academic context surrounding these questions and the implications of the results. While readers see the end product, perhaps as a succinctly stated theorem, they are often uninformed of the numerous refinements to definitions and lemmas that math research usually involves. Essentially, mathematical papers tend to obfuscate the process of generating mathematical content, as well as the context where their research is situated, in favour of presenting the results of research as an idealized, unified whole.

Another aspect of mathematical writing Csizsar focuses on is the approach to mathematical rigor. Although we may think of math as the standard of rigor, most published proofs are not, in fact, logically complete. In practice, it is often recommended to omit calculations that are regarded as "routine" or "standard", and instead focus on things like "unexpected tricks". While most mathematicians may have an idea of what is "routine", it's worth questioning what these terms actually mean, and how they might mean different things to different readers. Csizsar speculates about the use of "discourse markers" that signal to the reader to fill in any gaps and act as "markers of trust", reassurances that the writers know what they're doing. This seems to imply that rigor as we understand it is less about absolute logical completeness and more about convincing rhetoric.

I found Csizsar's article to be an enlightening read, and it led me to ask myself how we can apply these observations to our own work. For one, it helps to be aware of the unconscious biases that guide our approach to writing; mathematical writing, like any genre of writing, comes with its own conventions, and it's important to understand these conventions as well as when to break them. For example, what details are worth including or omitting, and why? What exactly is "standard" about this particular calculation? What context or motivation might help to frame the argument or results being presented? Being aware of audience and genre helps us make more informed decisions in our writing.

Acknowledging unspoken conventions can also help guide our reading, which can also improve our writing. Moving away from the assumption that mathematical writing is strictly objective, we can ask questions about what choices are being made in the papers we read. What context is being omitted? What motivated the research in the first place? How might this proof be different if it was presented for the purpose of teaching, as opposed to appearing in a journal? What aspects of the writing, such as organization or word choice, do you like or dislike? Analyzing the text as a piece of writing and considering authorial decisions and intent — in a sense, approaching it from the perspective of literary analysis — can inform our own writing and open our eyes to the power of language as a means of communicating difficult ideas. □

Puzzles

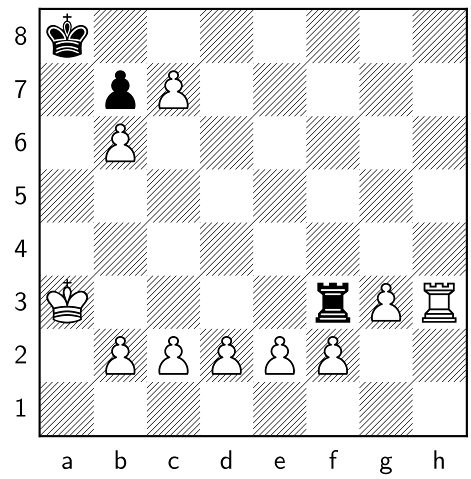
by Matthew Bolan

Math Puzzle

You probably know that $S_k \cong \mathrm{Aut}(S_k)$ when $k \notin \{2, 6\}$, but is there a non-trivial abelian group G with $G \cong \mathrm{Aut}(G)$?

Chess Puzzle

This puzzle by Otto Gallischek is one of my favorites, and quite unlike normal chess puzzles. It first appeared in the newspaper Weser-Kurier on February 25th, 1960.

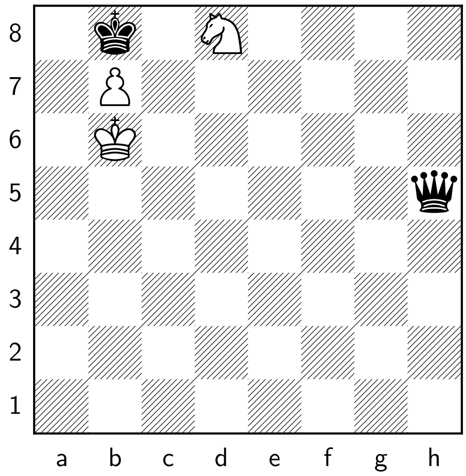


Black has just played 1...♖f3+, with the intent of sacrificing the rook to force a draw by stalemate. Can White stop this?

Good luck! E-mail all solutions to mgsa [at] studentorg [dot] utoronto [dot] ca

Solutions for Previous Puzzles

A Strange Endgame



n^{th} Power Groups

The Infinite Case

A group that works in the infinite case is $G = \langle x_0, x_1, x_2, ... | x_0^k \rangle$, the free group on countably infinitely many generators times a cyclic group of order k with generator x_0. Crucially, this group is countable as the set of finite sequences of naturals is countable, so we may choose some bijection $f : \mathbb{N} \to G$ with $f(0) = e$. Let $g_i = f(i)$. We now inductively build finite sets $\{A_i\}_{i=0}^{\omega}$ such that $A_i \subseteq A_{i+1}$, g_i is a product of k elements of A_i, and every element of G is a product of k elements of A_i in at most one way.

For A_0 we may take $\{x_0\}$. Now assume we have already built A_{i-1} and wish to build A_i. If g_i is already a product of k elements of A_{i-1} we are done, so suppose this is not so. As A_{i-1} is finite, we may select $k-1$ generators $x_{j_1}, x_{j_2}, ..., x_{j_{k-1}}$ which do not yet appear in any element of A_{i-1}, and let $A_i = \{x_{j_1}, x_{j_2}, ..., x_{j_{k-1}}, (x_{j_1}x_{j_2}, ...x_{j_{k-1}})^{-1}g_i\} \cup A_{i-1}$. Clearly this remains finite and g_i is a product of the k new elements. A slightly tedious check using the freshness of the x_{j_s} as well as $g_i \neq e$ and $e \notin A_i$ confirms that no two k-fold products of elements in A_i can agree.

Finally, taking $A = \bigcup_{n=0}^{\infty} A_n$ finishes, as every g_i is a product of k elements in $A_i \subseteq A$, and any failure of uniqueness of this representation would be contained in A_m for some finite m.

The Finite Case

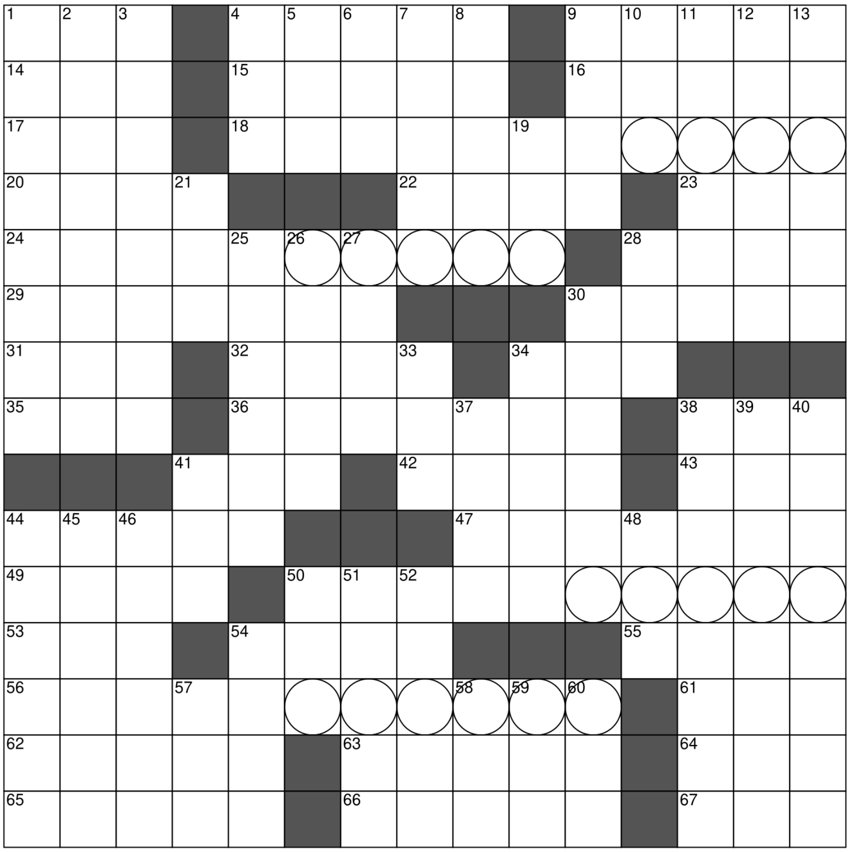
The following proof is due to Andy Jiang. A proof through a more graph theoretic lens can be found in Donald Knuth's *Notes on Central Groupoids*.[1] For the finite case, suppose for sake of contradiction G is a non-trivial finite group and $A \subseteq G$ is such that every element $g \in G$ is expressible as a product of $k > 1$ elements of A in a unique way. Now consider the group algebra $\mathbb{C}[G]$, which we decompose into the trivial representation $S = \{a \sum_{g \in G} g | a \in \mathbb{C}\}$ and its orthogonal complement $S^{\perp} = \{\sum_{g \in G} a_g g | a_g \in \mathbb{C}, \sum_{g \in G} a_g = 0\}$. Left multiplication by the element $\eta = \sum_{g \in A} g$ preserves S and $S^{\perp}$, and acts as multiplication by $|A|$ on the one dimensional space S. Furthermore, since $\eta^k = \sum_{g \in G} g$ annihilates $S^{\perp}$, left multiplication by η is nilpotent and thus trace 0 on $S^{\perp}$. Combining, we see that left multiplication by η has trace $|A|$. As elements of G act by left multiplication with trace either 0 or $|G| > |A|$, this means η cannot be a sum of elements of G, a contradiction. $\square$

[1] Donald E. Knuth. Notes on central groupoids. In: Journal of Combinatorial Theory 8.4 (1970), pp. 376–390. https://www.sciencedirect.com/science/article/pii/S0021980070800321

Crossword

by Kevin Santos

I don’t want to spoil anything about the theme for this crossword, other than the fact that it’s math-related, but take note of the circled letters as you fill in the grid. I hope that you enjoy figuring out how the theme answers at 8-, 24-, 50-, and 56-Across relate to each other, which will help you fill in the answer at 36-Across. I tried to make this puzzle accessible for beginners, so don’t worry if you have less experience with crosswords. Happy puzzling!



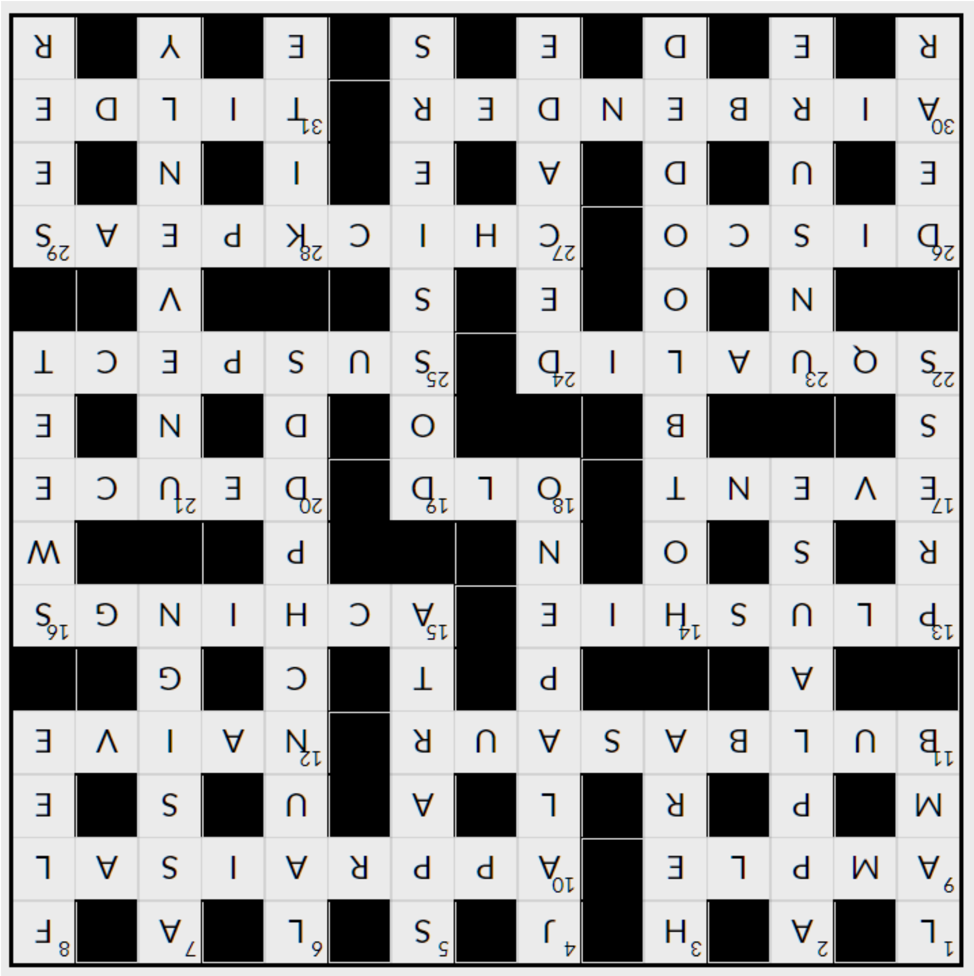
ACROSS

1 It follows Nov.
4 Daughter of Mummy and Daddy Pig
9 Four-door car, e.g.
14 Prefix with skeleton or planet
15 Flicked the residue off of, as a cigarette
16 Adler who outwitted Sherlock Holmes
17 Musical releases longer than singles but shorter than albums
18 It's said to encircle the "vein of love"
20 It may be guilty
22 Horse rider's strap
23 Org. offering roadside assistance
24 Staple in market research
28 Word with Bible, asteroid, or green
29 "To the stars", in Latin
30 For better or for _____
31 Barbie's partner
32 Fruit peel
34 "Hallowed be ___ name"
35 Ontario's time zone
36 Domain where the circled letters can be found
38 Texter's "Whatever"
41 Bird that can rotate its head 270 degrees
42 ____ one's own horn
43 American gun lobby org.
44 Collect
47 Intrinsic, like certain functions in a programming language
49 Summon
50 Sheaf origin?
53 Symbol for multiplicative identity
54 Apple product discontinued in 2022
55 Word aptly found in the letters of "unsightly"
56 Spacecraft used in certain 44-Down missions
61 Subside
62 Set free
63 Spherical pink Nintendo character
64 Coastal feature whose name originates from the Spanish for "river"
65 Mary-Kate and Ashley's surname
66 Intuition
67 Data type consisting of a series of characters (Abbr.)

DOWN

1 Video created with artificial intelligence
2 Blows up
3 Certain trigonometric function
4 Feline foot
5 Direction opposite WNW
6 Goal of many graduate students
7 Actor Pascal
8 French exit?
9 Common type of error in a calculation
10 Mess up
11 Must-have for Canadian winter driving
12 Prestigious mathematics journal, informally
13 What "¬" is sometimes used to do
19 Small bite
21 Canberra's country (Abbr.)
25 Supply at a 37-Down shop
26 Question relentlessly
27 Called
28 Youngster
30 "Consider this scenario..."
33 Certain multilinear alternating function (Abbr.)
34 Rainbow fish
37 Chatime offering
38 Set represented by Z for Zahlen
39 Tool part used to bore holes
40 Coffee Crisp, e.g.
41 Carbohydrate suffix
44 NASA program that put a man on the moon
45 Lin ______ Miranda, writer of *Hamilton*
46 FBI operatives
48 Lucy of *Charlie's Angels*
50 Stat that can be measured at typing.com
51 Catchy parts of songs
52 Comedian Murphy
54 With 57-Down, prehistorical period seeing advancements in metallurgy
57 See 54-Down
58 Vessel often used for ashes
59 Abbr. on a scale
60 Potato spot

Solution to the Previous Crossword



Two Questions with Alain Connes

by Waleed Qaisar

This is an excerpt from an interview with Alain Connes. It has been edited for clarity and conciseness.

Today we're at the Fields Institute to interview Alain Connes. Alain, of course, needs no introduction. He's the recipient of the 1982 Fields Medal, along with a host of other awards. His work has been important in the theory of operator algebras and noncommutative geometry, and he has fashioned new powerful methods in theoretical physics. He has also used these ideas to fashion an extremely novel approach towards the Riemann hypothesis.

For the audience, I want to say beforehand that this will not be a typical interview, as several of these have been conducted with Alain before and are available online. What I wanted to do here is to provide freedom for him to go into as much depth as he wants about a particular topic, and also to allow the freedom for sharp changes of topic. So it might be that the interview hangs together less coherently than a traditional interview, but still, we should have some fun. So we'll get right into it.

Qaisar: Thank you for being with us here today, Alain. The first thing that I was going to ask for your opinion on is, when doing mathematics, how may one detect that a change of language is fertile or will actually bring one closer to the resolution of a problem, and when instead it might just be a dead reformulation of a problem - what gives one the courage to commit fully to an idea or to a different approach? Beforehand, before knowing.

Connes: I would like to take some examples. The first example which I will take is not really mathematics, it's really theoretical physics. It is what happened when Heisenberg really discovered matrix mechanics, but somehow he didn't know that he was dealing with matrices. And I mean, so one could argue that, when Born and Jordan reformulated what Heisenberg was doing in terms of matrices, they just reformulated it. And one could say, okay, but all the job was done by Heisenberg. So these guys, by reformulating it, didn't do much. But in fact, this would be a big mistake, because, you know, after what happened with Born and Jordan, then I think it was Hilbert who asked around, you know, how you mathematicians must find a way to understand what physicists are doing. And von Neumann was around the corner. And this is a time when von Neumann created the Hilbert space as a fundamental stage for operators, for quantum mechanics and all that. So, I mean, it's typical of a situation in which, if one was very, "Okay, well, this guy didn't do much because they just reformulated things" and so on, you wouldn't get the point at all. Now, there are many other examples.

There is the other example which I have in mind, the example of schemes, you know. So when you do algebraic geometry, if you are sort of very down to earth, you can say, well, I have my algebraic equations, and I don't care any about anything else, you know. I mean, that's it. And so then when you see that the reformulations of, you know, what happens with schemes and all that, you might be a little bit negative and say and assert, you know, that this is just a reformulation of things that we knew before. That is partly true, of course, but what you find out in the long run is that this is the right point of view, but it takes time. It takes a lot of time.

And then your question says, you know, what does it mean to believe in an idea? Well, okay, I mean this is something extremely personal. I mean it is this what one would like to call intuition, you know, namely, that you have a sort of gut feeling that you know this idea is the right one. And so then you test it, of course, against a lot of examples and things like that. And you know you get more and more confidence in it, and so on. And this is something which is very difficult to transmit. I mean this is something which cannot be - I mean, okay, one can try to transmit it orally, and so on. But there is this kind of very conservative point of view, which is, people tell you, "Okay, you have an idea, but what is the theorem?", you know? And I think this is a very negative point of view, actually. I don't like this point of view at all, because an idea, when it's born, you have to protect it, and you have to let it grow by itself. And if you try to kill it from the start, some people, for instance, are very fast. So they would tell you, okay, but this won't work in this case. Don't listen to them - keep your idea and try to, you know, to make it grow and so on, but protecting it at the beginning.

Q: You have said previously you have to preserve your own ignorance. How much do you - over your career, have you looked at the literature a lot? Or do you find that looking at the literature too much kills creativity?

C: In my case, I only communicate by talking. I never look at the literature, essentially.

I mean, okay, first of all one has to work in a domain with where you know more or less where the boundary is. So, this is fine, but what I have in mind is that usually, for instance, if I look at the book, I much prefer to start by the end of the book, look at the theorem, and then close the book and try to think if I can prove this theorem. And I know for myself - I mean, every mathematician has a specific way of functioning - but my own way of functioning is that if I would open a book and I read it, I would have forgotten the whole book the next day or the next week. But if I pick a theorem, and, "My God, I cannot do it!" And I think about it, and "Ahh I cannot do it!", and, you know, and I am old and all that, and okay, perhaps after a few days, two possibilities: either I found my own way, or I didn't. But if I didn't and I look in the book, well, immediately I will spot the place where something is going on. Immediately. So whereas otherwise I would read, and I would read line by line, and I would get nothing.

So that's my way of functioning. It's more like I like to find the statement which *I* consider very meaningful, but which I cannot prove. And if then I am able to think about it and perhaps approach it and so on, then it stays in my mind forever. If I applied enough effort, of course - you know, you have to apply enough effort.

But there is another way in which I function sometimes, which is to take a statement for granted. You know, there is a statement. This statement is so beautiful and so on. You know that if you are going to think about it and so on, you are going to spend too much time, so you take it for granted, put it in your pocket. That also works, actually, in some cases. But it is very very rare.

Q: I actually have been quite influenced by this. You've made this comment before: where you you look at the end of the book, and then you go for a walk.

C: Yeah, yeah, exactly. And what I would add to that is that you know, you would think that if the statement is very complicated, it's useless to go for a walk. No! Because you see, the main difficulty, I think, is that your brain is going to build a mental picture. And this mental picture will be built only if you are not in front of a piece of paper with a pen, and only if you are by yourself. I mean your brain by *itself*, which means taking a walk, because otherwise you are going to look at what you have written and you think that what you have written is in your brain. No, it's not at all. It only enters your brain when you are walking by yourself and you are trying to figure out, even, what is the problem? You see, it could be a very complicated calculation. Doesn't matter. Take it with you. Take it with you. And slowly the mental picture will be built. And once the mental picture is built. Wow! Then it lives by itself. It can wake you up during the night, you know, it can tell you maybe this is connected to that. But if it's on a piece of paper, nothing. On a piece of paper, nothing, nothing. Doesn't help at all. It has to be in your brain. And this is the most difficult part, to put it in your brain. Put it in your brain, not on a piece of paper. Piece of paper is useless because it's not in your memory. It's on a piece of paper, or on the computer, or in a book, that's useless. Until it's in your brain. □

...(asking for a friend.)

an anonymous Q&A column

Do you have a question about navigating the grad student journey that might be too embarrassing to ask? If yes, this column is for you! Each issue we'll collect anonymous questions from you readers, and collect your (short) responses to them too. You can ask fun questions too!

Here are the questions for this issue. Tell us what you think about any of them, and send your own questions to us using this survey.

- Did anyone pick up a new hobby during grad school?
- What helps you come up with your own original ideas for research?

Responses to questions from the January 2025 issue

Q: How do you know when to give up on a research problem?

Does changing the research area count as "giving up on a research problem"? If so, I realized I didn't like it that much, and I took it as "homework." I couldn't imagine my takeaway and future directions if I was done with that problem. At that point, I figured out my taste and what mathematics meant to me and developed other preferences instead of what I was working on.

Anonymous student

Of course, there is no direct answer for this. This is a question which depends on a lot of factors: How hard is the problem? How interested are you in the problem? Do you have the mathematical background for tackling the problem? With these and various other factors clouding our judgement, one of the key pieces of advice is to listen to your gut. If we are feeling strong resistance to working on a project, this may be an indication that something has to change. This could involve giving up on the problem, taking a break, or trying a new method.

In this situation, it is best to have an open conversation with your advisor(s). As graduate students, it is not expected of us to have foresight of what problems are easy and what approaches should be fruitful. That is the job of our advisors! It can look like a simple conversation with them regarding how you are feeling about your project, and what they think about the situation. Having regular communication with leading mathematicians is an excellent resource we have access to in our advisors, and there is lots for us to gain by taking full advantage of their judgement.

Newsletter Editor

Q: What is the spookiest experience you've had on campus?

Writing final exams. Incredibly spooky.

Anonymous student

Q: Do you ever worry about not being smart enough?

I felt like an imposter during my first year; everybody seemed smarter than me. I still do this occasionally, but it doesn't bother me that much now.

Anonymous student

Yes all the time, but I try to think of them as opportunities for growth. It's difficult though for sure. It also is related to the Genius and "Scenius" article, where in the math community and the society in general there's a myth about really smart people who know the answer without even thinking, haha. I think the truth is that everyone faces a lot of challenges.

Newsletter Editor

How to Contribute

Do you have some cool math to explain, or some art or craft to show us? Do you have someone you'd like to interview? Would you fancy a chance to dive into a non-math topic by writing a short essay about it? Send your potential ideas to mgsa [at] studentorg [dot] utoronto [dot] ca, and one of our editors will be in touch. No commitment required at this stage— let's just talk and see if anything cool emerges! You can also help us by giving feedback on this issue at this survey.

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