MAT 363S – Assignment #1

Due in class on Wednesday, Jan 21, 2004

Problem #1: [4 points] Calculate the arclength $s(t)$ for the spiral $\gamma(t) = e^{-t}(\cos t, \sin t)$, starting at $t_0 = 0$.

Problem #2: [12 points] Let $\gamma : (a, b) \to \mathbb{R}^2$ be a regular curve, and let $\beta(t)$ denote its evolute (=caustic).

a) Show that the tangent line to β at t equals the normal line to γ at t.

b) Find the signed curvature, and the equation for the evolute, for the catenary $\gamma(t) = (t, \cosh(t))$.

c) Find the signed curvature, and the equation for the evolute, for the ellipse

$$\gamma(t) = (a\cos(t), b\sin(t)).$$

with $a, b \neq 0$. Give a sketch of the ellipse and its evolute for $a = 4, b = 1$.

Problem #3: [4 points] One often describes a plane curve in polar coordinates,

$$\gamma(\theta) = R(\theta)(\cos\theta, \sin\theta),$$

where the angle $a < \theta < b$ is used as a parameter. Give a formula for the signed curvature $\kappa_s(\theta)$ in terms of the function $R(\theta)$ and its derivatives.