MAT 363S – Assignment #6

Due in class on April 2, 2004

Remark: We use the terminology "zero" and "index" for vector fields on surfaces, while our textbook prefers "stationary point" and "multiplicity".

Problem #1:[6 points] Let $\phi, \psi : S \to \tilde{S} \subset \mathbb{R}^3$ be two isometries between regular connected surfaces. Assume that for some point $p \in S$, $\phi(p) = \psi(p)$, and also $\mathrm{d}_p\phi(\xi) = \mathrm{d}_p\psi(\xi)$ for all $\xi \in T_pS$. Show that $\phi = \psi$ everywhere. (Hint: Argue with the exponential map.)

Problem #2:[8 points] Let $S \subset \mathbb{R}^3$ be a compact oriented surface, and $p \in S$. Let ξ be a vector field on S, with an isolated zero at p.

a) Can it happen that the index of ξ at p is 0? Justify your answer.

b) Can it happen that $\xi \neq 0$ everywhere on $S \backslash \{p\}$? Justify your answer.

Problem #3:[6 points] Let $S \subset \mathbb{R}^3$ be a regular oriented surface, and $p \in S$. Let $\epsilon > 0$ be sufficiently small, so that the exponential map $\exp_p : B_\epsilon(0) \to S$ is a well-defined diffeomorphism onto its image. (Here $B_\epsilon(0)$ denotes the open disk of radius ϵ in T_pS.) Let u, v be the local coordinates on S, induced by a choice of orthonormal basis in T_pS. Show that in these coordinates, all of the Christoffel symbols vanish at $(u, v) = (0, 0)$.