MAT 363S – Assignment #4

Due in class on March 8, 2004

Problem #1: [5 points] Let $S \subset \mathbb{R}^3$ be an oriented surface, and $p \in S$. Fix $v \in T_pS$. Let $\kappa_n(\theta)$ denote the normal curvature at p of any curve $\gamma(t)$ in S, with $\gamma(0) = p$, and such that $\dot{\gamma}(0)$ makes an angle θ with v. Show that the mean curvature of S at p is given by

$$H(p) = \frac{1}{\pi} \int_0^\pi \kappa_n(\theta) \mathrm{d}\theta.$$

Problem #2: [8 points] Calculate the Gauss curvature $K(u, v)$ for the Moebius strip with parametrization,

$$\sigma(u, v) = \xi(u) + v(\cos(u/2)\zeta + \sin(u/2)\xi(u)),$$

where

$$\xi(u) = (\cos(u), \sin(u), 0),\ \ \eta(u) = (-\sin(u), \cos(u), 0),\ \ \zeta = (0, 0, 1).$$

Problem #3: [7 points] Let $S \subset \mathbb{R}^3$ be an oriented surface, with unit normal vector field $\mathfrak{N}$, and assume $K < 0$ everywhere. Suppose that for any two intersecting curves $\gamma_1(t), \gamma_2(t)$ on S, the angle of intersection on S coincides (up to a sign) with the angle of intersection in S^2 of their images $\mathfrak{N} \circ \gamma_i$. Show that $H = 0$.