MAT 267F – Assignment #1

I am hoping that Crowdmark will become available for this course next week. For the time being, here are the problems so that you can already start working on them.

Problem #1: For the following 1st order ODE's, sketch some isoclines (indicate which slope m they correspond to), the direction field, and some sample solution curves, using different colors for the isoclines and solution curves. (You don't have to actually solve the equation.)

a)
$$y' = \frac{1}{y}\sin(x).$$

b)
$$y' = \frac{x+y}{x-y}.$$

Note: You will be able to double-check your solution with an online program. However, please work out the solution yourself, and draw the picture by hand.

Problem #2: Find the general solutions of the following ODE's.

a)
$$y = xy' - \sqrt{x^2+y^2}$$

b)
$$y' + y = 2xe^{-x} + x^2$$

c)
$$(x^3+y^3)\ dx - xy^2\ dy = 0$$

d)
$$xy' + y = x^4y^3.$$

Problem #3: By making a substitution of the form $y = ux^n$, show that the following ODE can be transformed into a separable equation, and thereby solve it.

.
$$\frac{dy}{dx} = \frac{1-xy^2}{2x^2y}.$$

Problem #4: Determine which of the following equations are exact (justify your claim!), and solve the ones that are.

1. $(x + \frac{y}{2})\ dy + y\ dx = 0$.
2. $(\sin x \tan y + 1)\ dx + \cos x \sec^2 y\ dy$.
3. $(y - x^3)\ dx + (x + y^3)\ dy = 0$.
4. $(2y^2 - 4x + 5)\ dx = (4 - 2y + 4xy)\ dy$.

Problem #5: a) Show that if y solves the **Riccati equation**

$$y' + g(x)y + h(x)y^2 = k(x)$$

then the function

$$u(x) = \exp\left(\int_{x_0}^{x} h(t)y(t)\ dt\right)$$

(for arbitrary choice of x_0) satisfies a *second order* linear homogeneous ODE.

Thus, the initial value problem $y(x_0) = y_0$ for the Riccati equation is equivalent to an initial value problem for this second order ODE, with initial conditions $u(x_0) = 1,\ u'(x_0) = h(x_0)y_0$.

b) Apply this method to solve the initial value problem

$$y' + y + e^x y^2 = -4e^{-x}, \qquad y(0) = 3.$$

(Here, the resulting second order equation will be simple enough so that you can solve it, even without knowing the general theory.)

Additional problems: You should practice some examples of the standard (separable, homogeneous, linear, exact, Bernoulli) ODE's, making sure that you know, at least in principle, how to solve them. See e.g. Tenenbaum Pollard, Exercise 7 (page 71), Exercise 9 (page 79), Exercise 10 (page 90), 11 (page 97). The 'miscalleneous' exercises on page 104 should be useful: Here, the main task is to see what type of equation it is (sometimes after substitution); once you see it you should just move on to the next problem, without spending time on the details.