Section L0101

Quiz #1 – Solutions

① $t\,y' + (1-t)\,y = t \qquad (t>0)$

$\Rightarrow \quad y' + \left(\frac{1}{t}-1\right)y = 1 \qquad$ (linear)

Integrating factor: $\mu(t) = \exp\left(\int\left(\frac{1}{t}-1\right)\right) = \exp(\ln(t)-t)$

$= t\,e^{-t}$

$$\frac{d}{dt}\left(t\,e^{-t}\,y\right) = t\,e^{-t}$$

$\Rightarrow \quad t\,e^{-t}\,y = \int t\,e^{-t}\,dt = -(t+1)\,e^{-t} + C$

$\Rightarrow \quad \underline{\underline{y(t) = -\left(1+\frac{1}{t}\right) + C\,\frac{e^{t}}{t}}}$

② $y' = \dfrac{t^2 - ty + y^2}{t^2}\,,\ t>0 \qquad$ homogeneous; put $v = \frac{y}{t} \Rightarrow v' = \frac{y'}{t} - \frac{y}{t^2}$

$$v' = \frac{1}{t}\left(y' - \frac{y}{t}\right) = \frac{1}{t}\left(1 - \frac{y}{t} + \frac{y^2}{t^2} - \frac{y}{t}\right) = \frac{1}{t}\left(1 - 2v + v^2\right)$$

$= \frac{1}{t}\,(1-v)^2 \qquad$ this is separable:

$$(1-v)^{-2}\,dv = t^{-1}\,dt$$

$\Rightarrow \quad (1-v)^{-1} = \ln(t) + C$

$\Rightarrow \quad v(t) = 1 - \dfrac{1}{\ln(t)+C}$

$\Rightarrow \quad \underline{\underline{y(t) = t - \dfrac{t}{\ln(t)+C}}}$