

Quiz #3 - Solutions (LO101)

1) Solve $y^{(4)} - y'' = t$

Characteristic polynomial: $r^4 - r^2 = r^2(r^2 - 1)$
has roots $r_1 = 1, r_2 = -1, r_3 = r_4 = 0$.

Corresp. fundamental set of solutions:

$$y_1 = e^t, y_2 = e^{-t}, y_3 = 1, y_4 = t.$$

By undetermined coeff's, $y = -\frac{1}{6}t^3$ solves the inhom. equation.

$\Rightarrow$ General solution $y = c_1 e^t + c_2 e^{-t} + c_3 + c_4 t - \frac{1}{6}t^3$

2) Solve $\vec{x}' = \begin{pmatrix} 3 & 4 \\ 1 & 3 \end{pmatrix} \vec{x}$

The eigenvalues and eigenvectors are:

$$r = 5 : \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \quad r = 1 : \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

$\Rightarrow$ unstable node

Solution:

$$\vec{x} = c_1 e^{5t} \begin{pmatrix} 2 \\ 1 \end{pmatrix} + c_2 e^t \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

