MAT 244 (Fall 2012) Section L0101

Quiz #2 – Solutions

1) $2y'' + 10y' + 17y = 0$; $y(0) = 3$, $y'(0) = -3$

Char. eqn $2r^2 + 10r + 17 = 0$ has roots

$$r_1, r_2 = -\frac{10}{2\cdot 2} \pm \frac{1}{2\cdot 2}\sqrt{10^2 - 4\cdot 2\cdot 17} = -\frac{5}{2} \pm i\,\frac{3}{2}$$

$\Rightarrow$ Gen. solution

$$y(t) = C_1 e^{-\frac{5}{2}t}\cos\left(\tfrac{3}{2}t\right) + C_2 e^{-\frac{5}{2}t}\sin\left(\tfrac{3}{2}t\right)$$

Get $y(0) = C_1$, $y'(0) = -\frac{5}{2}C_1 + \frac{3}{2}C_2$

Hence, initial conditions give $C_1 = 3$, $C_2 = 3$.

Final answer:

$$y(t) = 3e^{-\frac{5}{2}t}\left(\cos\left(\tfrac{3}{2}t\right) + \sin\left(\tfrac{3}{2}t\right)\right)$$

(Can also be written as

$$y(t) = 3\sqrt{2}\, e^{-\frac{5}{2}t}\cos\left(\tfrac{3}{2}t - \tfrac{\pi}{4}\right).\ )$$

2) $y'' + 6y' + 9y = 0$

Char. eqn $r^2 + 6r + 9 = 0$ has repeated roots $r = -3$.

$\Rightarrow y_1(t) = e^{-3t}$, $y_2(t) = t e^{-3t}$

are solutions. Their Wronskian is

$$W[y_1, y_2](t) = \begin{vmatrix} e^{-3t} & t e^{-3t} \\ -3e^{-3t} & (1-3t)e^{-3t} \end{vmatrix}$$

$$= \underline{\underline{e^{-6t}}}$$