Review of Villani's "Birth of a theorem"*

Robert J McCann†

Abstract

Cédric Villani's newly translated popular work "Birth of$_a$ Theorem: $\frac{a}{m}_{athematical}$ Adv(e^n)ture" is a book without precedent. It was apparently born of an encounter with Olivier Nora, who suggested to Villani that, in the wake his 2010 Fields Medal, the general public would be fascinated to have the opportunity to read anything accessible he might be able to write on the nature of his award-winning work. Here work means not only the mathematical content of what was produced, but also the process by which it was achieved, and indeed the daily life and modus operandi of a leading mathematician. In response to this challenge, Villani offers the reader a window into the three years of his life during which he collaborated with Clement Mouhot on the problem of Landau damping. His account turns out to be equal parts diary, documentary, collage, stream of consciousness, and mathematical history, biography and exposition. As the translator notes: "The book is meant chiefly as a work of literary imagination... The technical material, though not actually irrelevant, is in any case inessential to the story..."

The story opens in the laboratory of the Ecole Normale Superiere de Lyon on a Sunday afternoon, both collaborators in Villani's office to discuss prospects for the Boltzmann equation (which describes the evolution of a colliding gas statistically). By the end of the conversation, several pages later, Villani has reconstructed a positivity result for a collisionless gas which he heard about from a postdoc in Princeton, and Mouhot has wondered aloud whether the argument might be relevant

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†Department of Mathematics, University of Toronto, Toronto, Ontario, Canada mccann@math.toronto.edu

to some controversies concerning the long-time behaviour of a charged plasma interacting electrodynamically. This is followed by a two page aside on Ludwig Boltzmann, in which his equation and his entropy are displayed, the latter observed to behave monotonically along the dynamics of the former, exhibiting one particular mathematical incarnation of the second law of thermodynamics. Its subsequent impact is described and some tragic details from Boltzmann's life are recalled. A depiction of Boltzmann is included, the first of a dozen or more sketches produced by Claude Gondard which tie together Villani's story and illustrate many of its principle characters, living and dead. It gives a sense of historical evolution in mathematics, great minds revisiting persistent questions century after century, building on each others insights to peel back layer after layer of some of the profoundest mysteries in science. (Why did structure emerge in the universe, and on what time scale?)

The second chapter flashes back to a lunch at Oberwolfach two years earlier, during which Villani's curiousity had been piqued by a conversation with a pair of experts about the Landau damping phenomenon in plasma physics. This concerned a paradox predicted by Landau: namely that Vlasov equation (which gives a statistical description of a Coulomb gas — either attractive in the case of gravity, or repulsive as in the case of electrically charged particles), despite being reversible in time, possesses certain equilibria which are stable, in the sense that the equation drives nearby initial data back to them both as $t \to +\infty$ and $t \to -\infty$! This damping phenomenon had been observed in many experiments, though it seemed to be violated in others. For a finite dimensional Hamiltonian system, conservation of the Liouville measure precludes such behaviour, but the Vlasov dynamics describe an infinite dimensional (statistical) limit.... Landau's analysis was based on a linearization of the dynamics. In a subsequent visit to Brown University, Mouhot had learned that the mathematical crux of the matter, which had never been resolved, was whether or not the nonlinear dynamics were correctly predicted by the linearization, and if so in what sense.

Their eventual resolution of this problem together was one of the principal contributions which garnered Villani his Fields medal. (For those who want to know the answer: they show the local damping effect is indeed produced in the nonlinear equation, but only for analytic or near-analytic initial data: for such data the dynamics smooths the spatial density while producing oscillations in the velocity variable; the convergence back to equilibrium occurs in a weak topology which doesn't see these oscillations.)

However, the book attempts rather to trace the actual, meandering trajectory which Villani traversed during the years he spent searching

for this result, the many random (and some less random) encounters which shaped his thinking on this problem. Certainly there are excursions into a variety of far-flung mathematical realms (including questions in number and graph theory, which have no relation to this problem), but there are equally many excursions to far-away conferences, attend childrens' music lessons, spend a semester at the Institute for Advanced Study, change jobs, explore the author's taste in poetry (e.g. William Blake), music (e.g. Catherine Ribiero), and books (e.g. Japanese comics). Also reproduced throughout the text are email correspondences from the period in question — many with his collaborator Mouhot, but others tangential to the story (including one from yours truly), a few pages from a math paper here, another there, excerpts from his book on optimal transport, a figure illustrating one of the reviewer's theorems, slides from a lecture on a different topic. The book is a charming insider's guide to a mathematician's world, meant to convey the mystery and the excitement which seduces us into the profession, the agony, the ecstasy, the quotidien. Designed to be accessible to outsiders, the original was a bestseller in France (titled "Théorème Vivant", meaning "Living (the) Theorem" or "Theorem Alive!"). Insiders will find it delightful as well. It offers them not only the chance to tag along with a future Field's medallist as he does some of his best work, but also an opportunity to provide friends and loved ones with a delightful glimpse into the foreign land in which we seem to spend so much of our time.