Fields Academy / MAT 1502H Prof. Robert McCann www.math.toronto.edu/mccann/1502

OPTIMAL TRANSPORTATION: GEOMETRY AND DYNAMICS

Lecture Hours: 13h30-15h00 Tuesdays and Thursdays (10 Jan - 8 April 2022, except for Feb 21-25)
Office Hours: Following class on Zoom or by email request

This course is an introduction to the active research areas surrounding optimal transportation and its deep connections to problems in geometry, physics, nonlinear partial differential equations, and machine learning. The basic problem is to find the most efficient structure linking two or more continuous distributions of mass — think of pairing a cloud of electrons with a cloud of positrons so as to minimize average distance to annihilation.

Applications include existence, uniqueness, and regularity of surfaces with prescribed Gauss curvature (the underlying PDE is Monge-Ampére), geometric inequalities with sharp constants, image processing, optimal decision making, long time asymptotics of dissipative systems, and the geometry of fluid motion (Euler's equation and approximations appropriate to atmospheric, oceanic, damped and porous medium flows). The course builds on a background in analysis, including measure theory, but will develop elements as needed from the calculus of variations, game theory, differential equations, fluid mechanics, physics, economics, and geometry. A particular goal will be to expose the developing theories of curvature and dimension in metric-measure geometry, which provide a framework for adapting powerful ideas from Riemannian and Lorentzian geometry to non-smooth settings which arise both naturally in applications, and as limits of smooth problems.

This course aims to help prepare graduate students for the Fall 2022 Fields thematic semester on *Nonsmooth Riemannian and Lorentzian Geometry*.

Prerequisite(s):

Corequisite: real analysis, e.g. comparable to U Toronto's MAT 1000-1001

References:

F Santambrogio. Optimal transport for applied mathematicians. Birkhauser 2015.

Ambrosio, Gigli and Savare: 'Gradient Flows in Metric Spaces and in the Space of Probability Measures'. Birkhauser 2005 (and subsequent works).

D Burago, Y Burago and S Ivanov "A course in metric geometry" AMS 2001.

Figalli: The Monge-Ampère equation and its applications. EMS 2017.

McCann and Guillen 'Five lectures on optimal transport' In Analysis and Geometry of Metric Measure Spaces G. Dafni et al, eds. Providence: Amer. Math. Soc. (2013) 145-180

Villani: 'Topics in Optimal Transportation', AMS GSM #58 2003

Villani 'Optimal Transport: Old and New', Springer-Verlag 2009.

GRADING SCHEME

20 % Attendance and participation (in person or by Zoom)
30 % Problem sets (roughly one per month)
50 % Written project (details below)

Late submissions will be penalized by 5% per day late.

NOTE THAT ALL STUDENTS NEED TO REGISTER THROUGH FIELDS AT
http://www.fields.utoronto.ca/activities/21-22/optimal-transportation
ZOOM LINKS WILL BE PROVIDED UPON REGISTRATION AT THIS SITE

PROJECT DESCRIPTION AND DEADLINES

5% Tuesday, Feb 15: Settle on a topic and references and communicate them to me by email
20% Tuesday, Mar 22: Complete draft project submitted for feedback
75% Thursday, Apr 7: Final draft report

This project is intended to provide you with an opportunity to independently pursue a related topic of interest to you within the framework of the course. Those taking this course for credit will be expected to prepare a written report on your topic (something on the order of 6-10 pages).

One possibility is to choose a paper, study it, and write a summary explaining its interest, importance and key contributions. In any case you should settle on a topic and format in consultation with me by end of January if possible and by Feb. 15 at latest. You are welcome to contact me at any time before that for project suggestions, or to shop your own proposals to me. Only the final draft will be graded, but this grade will be reduced by up to the indicated proportions if the preliminary deadlines (Feb 15, Mar 22) are not met.

This course is jointly offered by the University of Toronto and the Fields Academy. It will be governed by the following University of Toronto policies:

Academic Integrity

All suspected cases of academic dishonesty will be investigated following procedures outlined in the Code of Behaviour on Academic Matters. If you have questions or concerns about what constitutes appropriate academic behaviour or appropriate research and citation methods, please reach out to me. Note that you are expected to seek out additional information on academic integrity from me or from other institutional resources (for example, the University of Toronto website on Academic Integrity:

`http://academicintegrity.utoronto.ca/`

Equity, Diversity and Inclusion

The University of Toronto is committed to equity, human rights and respect for diversity. All members of the learning environment in this course should strive to create an atmosphere of mutual respect where all members of our community can express themselves, engage with each other, and respect one another's differences. U of T does not condone discrimination or harassment against any persons or communities.

Recordings of Lectures

The Fields Institute will be recording all lectures, including your participation, on video and making it available to the general public, including students in the course for viewing remotely and after each session. If you do not wish to appear in the recording, please switch off your camera before asking a question. Course videos and materials belong to your instructor, the University, and/or the Fields Institute depending on the specific facts of each situation and are protected by copyright. In this course, you are permitted to download session videos and materials for your own academic use, but you should not copy, share, or use them for any other purpose without the explicit permission of the instructor of the Fields Institute. For questions about the recording and use of videos in which you appear, please contact your instructor.

Accommodation of disabilities

The University of Toronto provides academic accommodations for students with disabilities in accordance with the terms of the Ontario Human Rights Code. This occurs through a collaborative process that acknowledges a collective obligation to develop an accessible learning environment

that both meets the needs of students and preserves the essential academic requirements of the University's courses and programs. Students with diverse learning styles and needs are welcome in this course. If you have a disability that may require accommodations, please feel free to approach me and/or the Accessibility Services office, and register at `https://studentlife.utoronto.ca/department/accessibility-services/`

Religious accommodation

The University provides reasonable accommodation of the needs of students who observe religious holy days other than those already accommodated by ordinary scheduling and statutory holidays. Students have a responsibility to alert members of the teaching staff in a timely fashion to upcoming religious observances and anticipated absences and instructors will make every reasonable effort to avoid scheduling tests, examinations or other compulsory activities at these times. Please reach out to me as early as possible to communicate any anticipated absences related to religious observances, and to discuss any possible related implications for course work.

Accommodation of illness or other emergency

If course work is delayed for a reason, the instructor has discretion to determine any remedy which he deems appropriate to the nature of the delay.

Family care responsibilities

The University of Toronto strives to provide a family-friendly environment. You may wish to inform me if you are a student with family responsibilities. If you are a student parent or have family responsibilities, you also may wish to visit the Family Care Office website at `family-care.utoronto.ca`