SOAR Math Course Homework Twelve Winter, 2002

These homework problems are meant to expand your understanding of what goes on during class. Any you turn in will be graded and returned to you. Answers may or may not be posted on the web, depending on demand.

Also, I've updated the cube solutions to fix the problem in step 6. The problem was that I had mistakenly used "B" when I meant "D." I'm very sorry.

1. Describe how to make a plus sign on four sides of a cube. That is, give a move (or set of several moves with instructions on what to do) that will get to this position from the solved state.

2. Describe the *two-slice* group. That is, consider the group generated by two slices (say, $\mathcal{U}$, the slice parallel to the Up side, and $\mathcal{R}$, the slice parallel to the Right side). Give a mathematical description of the group $\langle \mathcal{U}, \mathcal{R} \rangle$. (Hint: remember that the "spots" pattern can be obtained by $\mathcal{U}\mathcal{R}\mathcal{U}^{-1}\mathcal{R}^{-1}$. What does that mean about the order of this element? Also, what is the effect of $(\mathcal{U}\mathcal{R})^4$? What does this mean about the order of $\mathcal{U}\mathcal{R}$?)

3. Recall that in class we determined that the order of the $3 \times 3 \times 3$ Rubik's cube group (the number of possible positions of the cube) was

$$\frac{1}{2} \cdot 8! \cdot 12! \cdot 3^{8-1} \cdot 2^{12-1} = 43,252,003,274,489,856,000 \approx 4.3 \times 10^{19}.$$

   (a) What is the order of $2 \times 2 \times 2$ cube group? This should be manageable: imagine your cube with only corners, no edges or centers. There is a subtlety: is it possible to switch only two corners on a $2 \times 2 \times 2$ cube?

   (b) What is the order of $4 \times 4 \times 4$ cube group? (I expect this will be difficult to do without playing some with this larger cube to become familiar with it.)

   I will bring both of these kinds of cubes to class next week.