SOAR Math Course | Homework Ten | Winter, 2002

These homework problems are meant to expand your understanding of what goes on during class. Any you turn in will be graded and returned to you. Answers may or may not be posted on the web, depending on demand.

1. Find the knot group of a trefoil knot using the Wirtinger presentation, and simplify it as follows:
   (a) Write down the knot group G. It should have three generators: call them a, b, and c.
   (b) Eliminate c so that your presentation is $G = \langle a, b \mid aba = bab \rangle$.
   (c) Let $x = ab$ and $y = bab$. Show that you may rewrite this presentation as $G = \langle x, y \mid x^3 = y^2 \rangle$.
2. For an element g in a group G, let

$$C(g) = \left\{ hgh^{-1} \mid h \in G \right\}$$

   be the *conjugacy class* of g. (Compare this to the un-question, "Problem" 3 of Homework 6.) Show that if g_1 and g_2 are distinct (different) elements of G, then $C(g_1)$ and $C(g_2)$ are disjoint (have no elements in common).
3. Recall from Homework 9 that the center of a group G is the set $Z(G)$ of elements of G that commute with every other element of the group:

$$Z(G) = \{h \in G \mid gh = hg \text{ for all } g \in G\}.$$

   Describe $C(g)$ if g is in the center $Z(G)$.
4. Participate in the contest! See the course web page. Deadline for entries: Monday, December 16th at midnight.