SOAR Math Course   Homework Nine   Winter, 2002

These homework problems are meant to expand your understanding of what goes on during class. Any you turn in will be graded and returned to you. Answers may or may not be posted on the web, depending on demand.

1. Let $\phi : G \to H$ be a homomorphism from a group G to another group H. Recall that in class we defined the following three sets:

   - the kernel $\ker \phi = \{g \in G \mid \phi(g) = 1_H\} \subset G$.
   - the image $\operatorname{im} \phi = \{\phi(g) \mid g \in G\} \subset H$.
   - the preimage of a subgroup $K \subset H$: $\phi^{-1}(K) = \{g \in G \mid \phi(g) \in K\}$.

   Prove that:

   (a) $\ker \phi$ is a subgroup of G,

   (b) $\operatorname{im} \phi$ is a subgroup of H, and

   (c) $\phi^{-1}(K)$ is a subgroup of G. (You need to use the fact that K is a group here.)

   (That is, show that, for example: $1_G \in \ker \phi$ (identity), if $g \in \ker \phi$ then $g^{-1} \in \ker \phi$ (inverses), and if g and $h \in \ker \phi$ then $gh \in \ker \phi$ (closure). You need not show associativity.)

2. Define the *center* of a group G to be the set $Z(G)$ of elements of G that commute with every other element of the group:

$$Z(G) = \{h \in G \mid gh = hg \text{ for all } g \in G\}.$$

   Prove that:

   (a) $1_G \in Z(G)$.

   (b) If $h \in Z(G)$ then $h^{-1} \in Z(G)$.

   (c) If $h_1, h_2 \in Z(G)$ then $h_1 h_2 \in Z(G)$.

   That is, show that $Z(G)$ is a subgroup of G.

3. Find the center of (a) D_3, and (b) D_4. (Hint: one of these is trivial. That is, one of these has center equal to $\{1\}$.)

4. Participate in the contest! See Homework 8, problem 3. New deadline: Friday, December 6th at noon.