Slide 2:

1) To find all values of x s.t. $\sum_{n=0}^{\infty} a_n x^n$,

find all x s.t. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| |x| < 1$

(for any such x, series converges by Ratio test)

find all x s.t. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| |x| > 1$

(for any such x, series diverges by Ratio test)

So Ratio test tells us: 1) $\forall x \in \left(-\underbrace{\frac{1}{\lim\left|\frac{a_{n+1}}{a_n}\right|}}_{R = \frac{1}{\lim\left|\frac{a_{n+1}}{a_n}\right|}}, \frac{1}{\lim\left|\frac{a_{n+1}}{a_n}\right|}\right)$

$\sum a_n x^n$ conv

2) $\forall x \notin (-R, R)$, the series diverges.

3) $\frac{1}{R} = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

$$\frac{1}{R'} = \lim_{n\to\infty} \frac{n+1}{n} \cdot \left|\frac{a_{n+1}}{a_n}\right|$$

$$= \left(\lim_{n\to\infty} \frac{n+1}{n}\right)\left(\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|\right)$$

$$= 1 \cdot \frac{1}{R}$$

$$\Rightarrow R = R'$$

In fact, radius of conv of $\sum_{n=2}^{\infty} n^2 a_n x^{n-2}$ is also R

would go to 0 slower.

Side 3

suppose $(-3, 3]$
or $[-3, 3)$

n

Since $\sum a_n 3^n$ conv absolutely

$\Rightarrow \sum |a_n| 3^n$ converges.

What can we say about $\sum a_n (-3)^n$?

notice $\sum |a_n (-3)^n|$

$= \sum |a_n| 3^n$ which conv.

So $\sum a_n (-3)^n$ converges absolutely

If $\sum a_n 3^n$ CC, then

then $R = 3$.

so interval of convergence has to be

$(-3, 3]$ or $[-3, 3]$

Slide 4

$$\sum_{n=1}^{\infty} \frac{n}{2^n} = ?$$

$$f(x) := \sum_{n=0}^{\infty} x^n \quad \text{for } |x| < 1$$

$$= \frac{1}{1-x} \qquad \text{converges on } (-1,1)$$

for $x \in (-1,1)$, $\frac{1}{(1-x)^2} = f'(x) = \sum_{n=1}^{\infty} n x^{n-1}$

$$\frac{x}{(1-x)^2} = x f'(x) = \sum_{n=1}^{\infty} n x^n$$

so at $x = \frac{1}{2}$, $\frac{1/2}{(1-1/2)^2} = \sum_{n=1}^{\infty} n \frac{1}{2^n}$

2 "

Slide 5

Def: f is approx for g of order n near a if

$$\lim_{x \to a} \frac{f(x) - g(x)}{(x-a)^n} = 0$$

for 1 : is it true

$$\lim_{n \to \infty} \frac{P_n - f}{(x-a)^n} = - \lim_{n \to \infty} \frac{f - P_n}{(x-a)^n} = 0$$

for 3 :

$$\lim_{x \to a} \frac{[f(x) - P_n(x)] \cdot (x-a)^n}{(x-a)^n}$$

$$= \lim_{x \to a} \left[\frac{f(x) - P_n(x)}{(x-a)^n} \right] \cdot \lim [x-a]^n$$

$$= 0 \cdot 0 = 0$$