MAT137 - Outroduction

- We are DONE!

- Please don't forget to complete the course evaluation.

Prove that e is irrational

We will prove that e is an irrational number!

Suppose for the sake of contradiction that $e = \frac{a}{b}$ where $a, b \in \mathbb{N}$.

- As a start, use the N^{th} Taylor polynomial of e^x to approximate e find an expression for the error. Argue that e never equals its approximation for any N.
- Estimate the error by a number that only depends on N.
- Find the contradiction. Hint: multiply both sides by something so that the left side is just the difference between two integers.

L'Hopital's Rule Sucks 2

Use Maclaurin series to compute these limits:

1. $$\lim_{x \to 0} \frac{6 \sin x - 6x + x^3}{x^5}$$
2. $$\lim_{x \to 0} \frac{\cos(2x) - e^{-2x^2}}{x^4}$$
3. $$\lim_{x \to 0} \frac{(\sin x - x)^3 x}{(\cos x - 1)^4 (e^x - 1)^2}$$

Estimation

I want to estimate these two numbers

$$A = \sin 1, \qquad B = \ln 0.9.$$

1. Use Taylor series to write A and B as infinite sums.
2. If you want to estimate A or B with a partial sum, the fastest way is to use different theorems for A and B. What are they?
3. Estimate B with an error smaller than 0.001.

Coincidence?

Prove this:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \qquad \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^2}{90} \qquad \sum_{n=1}^{\infty} \frac{1}{n^6} = \frac{\pi^2}{945}$$

Hint: You know you can factor a polynomial if you are given its zeroes. You also know all the zeroes of $\sin x$. And you ALSO know that $\sin x$ is analytic so you can treat it as a polynomial.

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The Taylor series for $x \cot x$ is

$$x \cot x = 1 - 2 \left[\frac{1}{6} x^2 + \frac{1}{90} x^4 + \frac{1}{945} x^6 + \cdots \right]$$

Is this a coincidence?!